

# Categories and General Algebraic Structures with Applications

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# Categories and General Algebraic Structures with Applications

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# Contents

B. Banaschewski and A.W. Hager	
Representation of $H\text{-}\mathrm{closed}$ monoreflections in archimedean $\ell\text{-}\mathrm{groups}$ with weak unit	1
S. Ebrahimi Atani, S. Dolati, M. Khoramdel, and M. Sedghi	
Total graph of a 0-distributive lattice	15
F. Lucatelli Nunes	
On lifting of biadjoints and lax algebras	29
A. Karimi Feizabadi, A.A. Estaji, and M. Robat Sarpoushi	
Pointfree topology version of image of real-valued continuous functions	59
D. Hofmann and C.D. Reis	
Convergence and quantale-enriched categories	77
R.A. Borzooei, F. Hosseini, and O. Zahiri	
Convex L-lattice subgroups in L-ordered groups	139

8 Contents