



Categories and General Algebraic Structures with Applications

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Dedicated to Prof. Themba Dube



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In the Name of God

Categories and General Algebraic Structures with Applications

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Aims and Scope:

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