An interview with Themba Andrew Dube (A TAD Interview)

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Commemorating Themba Dube on his 65th birthday

Abstract. The paper covers an interview with Professor Themba Andrew Dube that captures his academic career that spans ordered algebraic structures, frames (locales), category theory, and pointfree function rings. The dialogue between the author and Dube features some of the work of Dube's doctoral students that influenced his research direction. The collaborative work of Dube and his relationships with prominent local and international scholars is exchanged in the conversation. We also narrate Dube's academic citizenry in Quaestiones Mathematicae, the journal of the South African Mathematical Society, whilst he was Editor-in-Chief. The paper gives a valuable historical account of Dube's contribution to the South African mathematical landscape and the African mathematical diaspora.

1   Prologue

This interview was conducted on 6 April 2023 at the trademark Pachas Restaurant, The Club Centre in Hazelwood (Tshwane), over Dube's favourites - a sumptuous Sirloin Togetherness and a special well-done T-Bone from the grill followed with coffee accompanied by dessert, a generous slice of baked cheesecake and a decadent caramel créme brûlée.   The transcript of the conversation post the luncheon was finalised via emails between the author (I.N.) and Themba Andrew Dube (T.D.).

2   The Interview

 I.N. *What made you change your field of study from Statistics to Pure Mathematics?*

T.D. Actually, what got me attracted towards pure mathematics at the University of Fort Hare (where I did my undergraduate studies up to master's degree) was an influence of my linear algebra lecturer who a year later was also my abstract algebra lecturer; and that was Loyiso Nongxa. The way that he handled abstract mathematics really appealed to me. I liked the rigour. Among my three major subjects I did Mathematical Statistics. There was some bit of what I viewed as somewhat hand-waving there. It did not have the type of rigour that I found in Pure Mathematics. So that's what really attracted me to Pure Mathematics, especially Algebra, Topology, Functional Analysis and so on.

Incidentally, when I registered for my Honours degree, apart from mathematics courses I wanted to take a course in Probability Theory and Stochastic Processes which was offered in the Statistics Department. At that time, as a student you had to negotiate with the other departments for joint subjects towards your honours programme and you could take at most two courses from other departments. The

Statistics professor, however, did not allow me to take a course in his department because he had tried to recruit me to do honours in Statistics, but then I had told him that I'd rather do Pure Mathematics. He was not available to negotiate so I ended up doing only Pure Mathematics courses. Now that I am retired, I will probably learn more probability theory during my spare time.

Then, when I did my master's degree, every Monday I had to travel from Fort Hare to Rhodes University (which is about 120 kilometres away) because I was attending lectures at Rhodes towards the qualification. The master's degree was by part coursework and a dissertation which was under the guidance of Nic Heideman at Rhodes. He at some stage ended up at the University of Cape Town. The dissertation was actually in Functional Analysis. Luckily, the University of Fort Hare (and I am very very grateful for that) paid for my travelling costs. The Dean's argument was: "We do not have the expertise at Fort Hare, you are registered with us, so let us carry the cost of ferrying you to Rhodes University to take the courses you require for your master's that we can't offer here". I took Topology and Functional Analysis at Rhodes University, and the others that I took at the University of Fort Hare were Algebra and Complex Analysis.

I.N. *How did you come to know about Pointfree Topology?*

T.D. I visited UCT (the University of Cape Town) to attend a conference in Statistics. At that stage I was teaching Statistics at the University of Zululand. They would not have funded me to attend a conference in any discipline other than Statistics. On one of the days of the conference I attended a lecture by Bernhard Banaschewski out of curiosity. A colleague had mentioned that he was visiting UCT and was presenting a seminar series. I had first met Bernhard Banaschewski when I was doing my Honours degree at the University of Fort Hare. He was invited by the South African Mathematical Society. When he was in South Africa at that time he was visiting some of the historically black universities and he did visit Fort Hare. He spoke to us about what he used to call a "hook function" which had something to do with calculus pitched at the level of undergraduate and honours students. So I had been impressed by the way he presented that lecture at Fort Hare.

If I may digress a little. Other mathematicians who came to Fort Hare when I was doing my honours were Serge Lang and László Fuchs, a Hungarian mathematician who has written some books in infinite Abelian groups.

Going back to the Banaschewski series of lectures at UCT, I attended one of those lectures and it turned out that he was introducing nearness frames at that stage. One bite and I was hooked. I saw that this is the kind of Topology that I really liked – with a decidedly algebraic flavour, and without "open sets" coming in to muddy the waters. That's how I got into pointfree topology. So it was purely through Banaschewski.

I.N. *How did you go about selecting an advisor for your doctoral studies?*

T.D. This was purely coincidental really. At that stage I was lecturing at the University of Zululand. So I wanted to do Topology, not necessarily Pointfree Topology. The then Chair of Department of Mathematics at the University of Zululand, Geoff Booth (who works in near rings) told me that some of the best names in Topology were those at the University of Durban-Westville (UDW). Actually there were two of them, and they were Dharmanand (Dharms) Baboolal and Ramesh Gungadeen Ori. So I made overtures to them and Dharms Baboolal said that I should come down (to UDW) for an interview, which I did. We had a discussion and then he (Baboolal) told me that he was getting into *frames.* He had spent about a year of sabbatical and had attended Banaschewski's lectures on frames at the University of Cape Town. So it turned out that I had already heard something about Pointfree Topology and so that's how I ended up with Baboolal. Actually Baboolal and Ori. I should point out Ramesh Ori was my co-supervisor.

I.N. *Incidentally, Ramesh Ori's doctoral advisor was Wolfgang Thron at the University of Colorado at Boulder* [44] *and Dharms Baboolal was supervised by Peter Collins at the University of Oxford towards his DPhil* [1]. *You spent some time at the University of Cambridge with Peter Johnstone. Can you give a brief account of your stay at Cambridge, your engagements with Peter Johnstone and any special memories you wish to share during that time?*

T.D. Yes. I received a grant from the British Council to spend what they called a *Lent Term* at Cambridge. At that stage I was still teaching at the University of Zululand. They said that I should write to somebody at Cambridge or Oxford or whatever good British university. I should make overtures to that particular person and ask them if they could host me for the Lent Term.

So I wrote a letter, that was pre-email days, to Dr Peter Johnstone and told him who I was, that I was Baboolal's student and that I had the grant from the British Council to come for the Lent Term. He replied and said that I was welcome to come over and get to work with him. But when I got to Cambridge, I thought that Johnstone was going to be doing something in frames and locales. He was actually running lectures in Topos Theory. Notwithstanding, I attended those lectures and I took notes. Alas, I am yet to read them (there's another "project" for my retirement.)

But you see, I was still working on my PhD at that stage. Johnstone said that I could always come and ask him questions about any topic. When I was looking at coproducts of nearness frames, for some reason I could not get to understand the structure. You know the nearness frame is a frame with something extra. I knew about coproducts of frames. So I went to him and asked him: *How could you put a nearness structure on this?* Then he suggested that I look at something and it turned out exactly to be the thing that I was looking for.

My first paper [3] from my PhD thesis [2] was on this coproduct of nearness frames. I used to keep Dharms abreast of what I was doing at Cambridge. Because I was so impressed with the results on co-product of nearness frames, I wrote to Dharms to assure him that the results were *my work*. Johnstone had guided me in a direction but the theorems were solely mine. I thought that I should do that lest my supervisor wondered how much of this was Johnstone's work.

I really enjoyed Peter Johnstone's lectures. I still have the hand written notes on Topos Theory although I have never really done any research on Topos Theory. At that time André Joyal was also at Cambridge and he used to come to Johnstone's lectures.

On the very first day of Johnstone's lectures, he (Peter Johnstone)

said that we should write down our names on a register. When the list came to me I saw the surname *Pretorius* – which is a fairly common South African surname. After the lecture, I asked one of the students who Pretorius was and he pointed him out to me. Indeed, he turned out to be a South African chap who actually did his PhD [45] under the supervision of Peter Johnstone. I used to go out to lunch with Pretorius.

It was during winter when I went to Cambridge in February. Coming from South Africa when it was midsummer, you can imagine! Very very hot. You go to this country, the sun has set around 2:20pm and so on. It's dark and nobody to chat with. There was, however, Pretorius who I started chatting with and he used to relate and talk about home and stuff. He was far advanced in his PhD and finishing at that stage.

I only spent about four months at Cambridge. I also met Martin Hyland at Cambridge. Johnstone mentions both Joyal and Hyland in his book [38]. I was fortunate to have met them both at Cambridge.

I.N. *Given the African diaspora, Africanisation, Decoloniality and Ethno-mathematics, are there any specific African mathematicians that have influenced your career?*

T.D. Not in the sense of the area in which they work in. But as I have mentioned at the beginning, I have been very much influenced by Loyiso Nongxa. When I went to Fort Hare, I did like mathematics. Actually, my parents wanted me to do pharmacy; however I was not into Chemistry and all those sort of things.

Then when Loyiso started teaching me at second year level, there was somebody I could identify with. He was doing mathematics really in the sense that would make you like mathematics. He was doing a Master's degree then. So when I finished my Honours, he had just finished his Master's at Fort Hare. I could see that one could go further than just a BSc and an Honours degree and you could further go into a Master's and a PhD. And the person that motivated me was an *African fellow*. Remember, that was during the peak of discrimination in South Africa. So you could identify with people of your kind, so to speak, especially if you came from the suppressed groups if I may

put it bluntly that way. So really, I was greatly influenced by Loyiso Nongxa.

I.N. *Loyiso Nongxa would further on graduate with a DPhil from the University of Oxford* [43] *and later on excel to become South Africa's first black Vice Chancellor of the University of the Witwatersrand. Are there any others going back to your undergraduate studies or further back in time? Sometimes we don't know about our African pure mathematics scholars at the time.*

T.D. No not really. Nobody else comes to mind.

I.N. *You have alluded to my next question. How did the previous apartheid governmental dispensation, prior to 1994, impact in your life as an up and coming mathematician and as an activist for a free South Africa? Were you arrested or imprisoned at any time for your involvement in the just course?*

T.D. Yes, that happened in 1977. I was a second year student at Fort Hare. On the day that Steve Biko was murdered (22 September), we demonstrated as students. So the police were called in. They told us to disperse as we had gathered at the university stadium. *We did not disperse.* So they arrested us and I spent four nights in prison. But luckily there were only students in the cells. They removed the hardcore criminals and put them separately and created cells only for students – we were flattered!. So I spent four nights in prison.

Another experience was during the first time I attended the South African Mathematical Society Conference. I was doing my Honours at the time and Professor Tom van Dyk, an Afrikaans fellow, tried to book me at the same hotel as he did for himself in Durban for the conference. They told him that they were not authorised to accommodate African people. That was a bit hurtful you know. But Tom van Dyk told me that these things do happen. That was something that *really* left a bad taste.

I.N. *Your initial work in your PhD covered nearness frames* [2]. *This was independent of the work of Banaschewski and Pultr at the time who were also working on pointfree nearness structures. You have constantly worked in the category of frames/locales particularly on*

compactness, pseudocompactness, Lindelöfness, paracompactness, re-
alcompactness, convergence of ideals and filters and structures on frames.
Something happened along the way that got you involved in rings.
What influenced this move and your research direction towards point-
free function rings and working with algebraic frames?

T.D. That was influenced by a doctoral student of mine - Oghenetega
"Tega" Ighedo. When Tega wrote and sent an email to me request-
ing that she'd like me to supervise her, I mentioned the topics that I
thought could be of interest to her. She said that she did know func-
tion rings and $C(X)$ and that she could learn about $\mathcal{R}L$. I had never
written a paper on $\mathcal{R}L$, rings of continuous functions on a frame. So
I said to Tega that these were rings that she could work on.

Pointfree function rings had been established already when Tega started,
but on the ideals thereof not much had been done. All that had been
done was by myself, I am to believe, actually looking at the maximal
ideals of these sort of rings. But other ideals and the ideal structure of
$\mathcal{R}L$ was virgin territory and not much had been done. So I asked Tega
if that was something that she would want to look on. It turned out,
of course, that she was coming into that area but not very familiar
with it. She said: "Whatever you recommend, rings yes I do know
and I have done some work on Commutative Algebra in my master's
degree and some Algebraic Topology". So we got into that.

When we were working on $\mathcal{R}L$, it turned out that some of the ideals of
that ring form what is called an algebraic frame. I had never written
a paper on algebraic frames. So I started getting into Jorge Martínez'
papers on algebraic frames because he was the one that had done lots
on that. So I learnt that very quickly and I directed Tega towards
that. She and I have subsequently written papers on those structures,
namely [17–24], by myself [7–13, 15, 16] and with some of my other
PhD students, Jissy Nsonde Nsayi [29–31], Lindiwe Sithole [32] and
Dorca Stephen [33].

So, its mainly an influence by some of my PhD students.

 I.N. *How did category theory come into your work and influence your re-*
search?

T.D. Around about 2010/2011 a fellow who used to work at the University

of the Witwatersrand, who works in Pointfree Topology, joined our department at UNISA. Prior to working with him, I had never done anything that's got category theory nor diagrams and all that. We then thought of a project that we could work on together. It required many commutative diagrams. I did not even know how they are drawn using LATEX and all that. The person is Inderasan Naidoo. In my first paper with him [25] all the diagrams were drawn by him. The mathematics was done by both of us. His inclination towards category theory affected that paper and subsequently influenced many of my other papers with him, some of mine alone, and with other researchers. So, in a nutshell then, I was driven towards a categorical look towards Pointfree Topology by working with Inderasan Naidoo.

I.N. *You have an admired student profile including Mugochi, Matlabyana, Ighedo and Nsonde Nsayi, to name a few. You have related about Ighedo's influence in your work. Have your other students influenced you into areas of research that you would not have thought about?*

T.D. Let me see. Not really. My first student was Martin Mugochi and he wanted to work on nearness frames [40]. I suppose having read my thesis [2] and some of my papers he just wanted to work on something that I had been working on. We simply worked on that.

Then came Mack Matlabyana. His inclination was more algebraic [39], and we did something on frames, extensions and all those sort of things. It was more something that I was working on, and then Tega came along. I have already spoken about the work with Tega [36].

Incidentally, when Jissy Nsonde Nsayi started his PhD, I had tried to get him to work on a certain field but it turned out that he was not much interested in that. He was interested in rings of continuous functions [42], which is something that I had already started working on and also with Tega. So there isn't much influence that I got directly from working with him.

Then I had a student called Dorca – yes, Dorca Stephen. Dorca came in and she had read something about *quantales*. So I said: *Okay, you like quantales and all that stuff, let's see how we can get into that.* Prior to that I had not written a paper that had quantales in it. Then working with Dorca, I just got deeper into the literature on quantales

and we had some papers with a quantalic flavour [33, 34] from her thesis [46].

Currently, I'm looking at localic coreflections of some quantales arising from ideals of function rings, something that I became aware of because of George Janelidze's talk at the conference (TACT2022) celebrating my retirement. So, I gathered that frames form a reflective and coreflective subcategory of the category of quantales, and currently I'm doing some work with regard to algebraic frames and quantales.

I.N. *So it's interesting that the two young ladies had an influence in your own research moving forward.*

T.D. Yes, it's purely coincidental that the influence was from my female students.

I.N. *You have had international collaborations with Banaschewski, Jan van Mill, Joanne Walters-Wayland and Papiya Bhattacharjee, to name a few, and locally, of course, with Naidoo and your students Mugochi and Ighedo amongst others. What advice would you give to upcoming graduate students on the value of collaborative work both locally and internationally?*

T.D. It's the sort of advice that I give to my own students. I'd tell them that once they have spent three or so years working with their supervisor, they should also look elsewhere. There is an influence upon you of the supervisor's way of doing things and the type of mathematics that he/she palates. When you read the literature you might discover that some other authors have certain ways of doing certain things. Make sure that when you go to conferences that you talk to those particular people. You can simply send them emails indicating that you've read their papers, that you are working on this particular area, and if it would be possible to collaboratively work with them. It is something that I always impress upon my students. Don't rely solely upon what you have learnt by way of being taught or by way of mimicking your supervisor.

Work with other people as well within your institution, within your country, within your continent and internationally. For instance, I have collaborated with some Iranians and in each of those instances it was them making overtures to me. All my Iranian collaborators are

much younger than me and they have indicated similarly that they have read my papers and wish to work with me. In some instances they have proposed the projects that have come to fruition collaboratively.

As you have mentioned, working with people like Bernhard, Papiya, Joanne and all those people, I have learnt certain things about doing Pointfree Topology using their styles of doing things.

I.N. *Which would you consider your best independent work and your best collaborative work thus far in your mathematical career?*

T.D. What I regard as my best independent work was my 2009 paper on maximal ideals of $\mathcal{R}L$ [6]. I'm not saying that because this paper seems to have received much more citations than any other. It's a paper that I really like. I also have a paper in Applied Categorical Structures [14]. It talks about pointfree disconnectivity with a ring theoretic slant. I also consider that as one of my nicest papers and I was glad to hear Jorge Picado say that he also liked that paper very much. That coming from a man of such stature says something. It's a paper in which I characterise these pointfree disconnectivities using *purely* ring theoretic methods.

My best collaborative work was with Joanne Walters-Wayland [35] which has got a very nice story behind it. It had been a practice with Joanne and I that at times she would have something and indicate that she had just submitted a paper and she would say to me: "I'm giving you a *privileged* preview". After I submitted the paper [5] I sent an email to Joanne indicating that I just submitted this particular paper. The following day she replied saying: "You know what Themba, I have been working on a similar thing. I have obtained some results which are like your ones. There are also some results that I've received that you have not received. How about we make this into a single paper?" I was of course agreeable to that. That paper [35] has also received quite a number of citations.

I.N. *You have published prolifically in what we, in South Africa, call accredited journals and in conference proceedings. Would you ever consider publishing a book or a book chapter or is there such in the pipeline?*

T.D. Yes, perhaps not a book chapter. Upon my retirement some of my colleagues, especially Inderasan Naidoo and Partha Pratim Ghosh, said perhaps I should consider writing a book on pointfree rings of continuous functions and their ideal structures. That is one of the projects I'm going to start on, now that I'm retired. It's something that I'm passionate about because a number of results have been obtained by myself, by the group in Europe with Jorge Picado, Javier Gutiérrez García and some of their students, including Imanol Mozo Carollo as well as some work by certain Iranians. The results are scattered over a number of papers. So, I would like to bring those together and also write more on ideal structures of the ring $\mathcal{R}L$. So, I am considering writing a book, not a book chapter as I have not thought about that.

I.N. *You have also held many senior administrative positions in your career ranging from Head of Department, Faculty of Science Dean and Deputy Vice Chancellor (DVC) of Academic Affairs and Research whilst you where at UniZul, the University of Zululand. You have also acted as Vice Chancellor of UniZul. How did these impact your research and would you advise faculty and graduate students on taking up these leadership roles in their career?*

T.D. When I was HoD, that did not impact much on my research. The amount of administrative work back then was not that overwhelming. I was dean for only two years, that did affect me a little because now suddenly you're managing faculties. You have to deal with heads of departments, some come from Zoology, Biology and all that. It's not the same kind of thinking as you are used to in the mathematical sciences. It takes a lot of reading to be able to understand what they are talking about.

When I was DVC of Academic Affairs and Research, that greatly impacted my research work simply because of the amount of meetings. The amount of having to sit down with people, university-wide for discussions on institutional matters did impact my research.

Would I advise budding researchers to take up these positions? Sincerely, I would say that if they are still just cutting their teeth into research and they are still building up their research profiles, then maybe they should not. Until they've built up a reputation as a re-

searcher and built up their CV, then maybe they can take up on that. If you do allow me, I would like to go back to some of my collaborative work.

I.N. *Of course please do, yes indeed.*

T.D. I'm thinking off the cuff here. There was a time that I was working with a colleague - Naidoo. From our first paper [25], we overlooked something and we made a subtle error. The error was not pointed out by anybody but we discovered the error when we were doing a subsequent paper. If you're working in the category of regular frames, your codense maps are precisely the injective ones. So we proved a result that overlooked that fact and that we were working with right adjoints. We acknowledged this and made the necessary amends in the corrigendum [26] which did not affect the essence of the work covered in [25].

Emanating from [26], we were able to prove a number of results including what I personally, I don't know about him, I would view as one of our finest papers. The paper that we (I say we but I should accredit him for the name) called *Round squares in the category of Frames* [28]. Reading that paper, one of the reviewers pointed out something about frame homomorphisms that both my coauthor and I had overlooked and I remember that vividly and it's very strange. It's that a frame homomorphism is injective if and only if its right adjoint is surjective.

Emanating from that then, my first visit to the US (United States of America) was at the invitation of Jorge Martínez, whom I've mentioned earlier. So I had travelled with Inderasan. That was my first flight out of Africa to the Americas. I had gone to some European countries before. In one of our visits from Chapman University to the City of Los Angeles, we proved results in the train that culminated in a paper [27] which was motivated by [26], our error in our original paper [25]. That's one of our papers [27] I really really enjoyed working on.

Coming from that, my student Ighedo and I wrote something that's got round squares in it [18]. From her thesis [36], she wrote a single authored paper [37] based on the ideas of [25, 26] and we together

wrote a paper on the ideas of the round squares [18]. With Ighedo as well, she has got something in her thesis [36, Chapter 5.2] about round squares. We (Naidoo and I) were hoping to get a student who had approached us from the University of Johannesburg (UJ) saying that she wanted to do some work with us and that's when we were busy with these round squares. So we gave her that project, but then she was employed at UJ and ended up not doing a doctoral studies with us but did a PhD in Functional Analysis at UJ.

So, that's also one of the influences from a collaboration with a younger colleague. That paper [28] was a very good paper Inderasan, it was very nice.

I.N. *You have the love for the cat and mouse challenge of Chess. Can you relate the work that you have done in the communities with Chess and its influence in students taking up mathematics?*

T.D. Chess! Yes. I live close to a township called Mamelodi. I live in Pretoria East and Mamelodi is about 17km from my house. So I used to play chess with the fellows there in Mamelodi. One of them is a chap called Hope Kubheka who is a maths teacher at his school (Mamelodi High). Somehow he became aware that I am a professor of mathematics. Then he said to me: "At some stage I would like for you to come and talk to my students, talk to them about mathematics, tell them about chess, as well" and he had already introduced chess at his school. So, I started doing that, and it was around 2014/2015 thereabouts and we were still at the Pretoria campus (reference to the premises of the Department of Mathematical Sciences at UNISA).

So I used to go there in Mamelodi (specifically for chess) and I viewed that as Community Engagement. But, alas, my university did not view that as Community Engagement. Community Engagement had to be a designated and approved college project. This was a single endeavour with a school and not affiliated to as a designated high school of the college that we could help or a college project. It was not viewed as Community Engagement. But I still did do it out of the love of mathematics and the love of chess. But with regards to the research students, I am not aware if any one of them plays chess. I am aware that one of my collaborators does play chess. We used to

play chess in an aeroplane at some stage when we were flying out of country. I do need to point that out.

I.N. *You are also very passionate about reggae music. How did this emanate in your life given the various musical genres and does it influence your mathematics in any way?*

T.D. Actually it does not influence my mathematics but there is something that I wrote in my master's dissertation. I thanked Loyiso Nongxa for having set an example and I thanked Bob Marley for having produced that music. Because at times I used to get frustrated, as all mathematicians do, I want to believe. You're trying to prove something and you just can't go about getting it, you know. You then want something just to relax and regroup for another assault on the result you want to prove, or a counterexample you want to construct. I started doing that when Bob Marley was very big. That was in the late 70's. You know, "Buffalo Soldier" and all those sort of things and I got into that music. The beat, the rhythm, his inimitably melodic voice, and all that used to calm me down. That's how I got into liking reggae music. I still do. I still do even now listen to a lot of reggae.

When this working from home came in, I could pace myself. In the mornings, I could clear my emails and try and prove some results. I would have lunch and then nap a little maybe for an hour or so. Then I'd listen to Burning Spear and other reggae stars. In the early afternoons I would start again with my mathematics.

My attraction towards reggae started whilst I was a third year student, it continued whilst I was an honours student, then into my master's, into my career as a lecturer, as a professor and even now really I still listen to reggae music. Incidentally, my darling daughter, Linda, really likes reggae music.

I.N. *You also have a rich interest in dialects and languages. Has this insight influenced your diversity and research into the history of mathematics?*

T.D. In the history of mathematics - not really. I'm very interested in language. My mother comes from one of the small ethnic groups in South Africa and speaks a certain language, one of the marginal languages - Venda.

When it comes to negation - e.g. I sleep, I do not sleep - you see now in English, the verb does not change - sleep, sleep and you say did not. I swim, I do not swim. In most South African languages, actually in all except Venda, when you negate, the verb also stays exactly the same. I don't know much about Afrikaans, I'm not talking about that, but the indigenous ones. Like when you say I sleep (for instance, in Zulu) - *ngiyalala*, and I do not sleep - *angilali*. The last vowel changes in all indigenous languages of South Africa, *except* in Venda. I once tried to research that.

When I joined UNISA, I spoke to a fellow in linguistics: *There's something that I've noted that in all these languages except Tshivenda, which is an indigenous language of South Africa, there is this thing about the verb not changing in negation. It's only in Venda that this does not happen. Pretty much in English. Is it something that we can work on?* Alas, he told me that I'm trying to bring too much mathematics into linguistics. I would rather not name the fellow.

You know they (the university) were trying to say that we must do interdisciplinary and intra-disciplinary research and all those sort of things. So I thought that I was bringing a lovely project to somebody in the humanities - somebody in linguistics. But this fellow told me my views seemed to be too much of mathematics than linguistics. So it ended up not being done.

I.N. *We know that Horst Herrlich wrote a few papers in general topology using the pseudonym Rhineghost. Have you ever written any mathematics papers under a nom de plume?*

T.D. I am very glad for that question. *I have never.* However, there was a stage when I was the editor of the journal QM (Quaestiones Mathematicae). There was somebody that sent a paper to QM. QM is a multidisciplinary journal in mathematics as you know. This person sent a paper in an area in which we had many submissions. So, I sent the paper to an associate editor that was closest to that particular area. The associate editor indicated that, because of the backlog, the paper was exactly in the sub-area in which we had many submissions and in which we were struggling to get referees, so he recommended that, in light of the huge backlog and the fact that the paper was not

exceptionally strong we should rather return it to the author.

As the Editor-in-Chief, I did not pass the buck. I communicated to the author that, in view of this, we were not going to consider the paper, and suggested that he should rather send the paper to a journal which specialises in his area. The author wrote back to me: "Dear Professor Dube. It really upsets me that you are not able to consider my paper. I'm editor as well of [he mentioned the journal's name] (*something I was aware of*). I might also be disinclined to consider papers in Pointfree Topology in our journal if I'm treated like this". I had to stand my ground. That was policy. Then some months later I wrote a paper that I believed would sit very nicely in his journal. I was tempted to write under a nom de plume, but I ended up not doing that and I sent the paper to another journal instead.

I.N. *So what about news articles in the public media, have you ever written such under a pseudonym?*

T.D. Oh yes. News articles I have. In the newspapers, yes I have. I used to write a lot to *The Star* newspaper – a once very reputable newspaper here in South Africa. There was a time that I wanted to write a letter, which I did, which was critical of the ANC government. I should say that openly. Critical of how the ANC government had gone off the rails. *The Star* was allowing that you could write under a pseudonym, but you had to give them your real name. So I have sent articles to *The Star* under a pseudonym. Yes, that I have done. But mathematics works - no. I was tempted, but I did not do it.

I.N. *How important is it being a referee and has this academic citizenry influenced your own research work?*

T.D. Refereeing papers has had a great influence in me. I have refereed papers that were sent to journals that I would normally not look at. I have become aware of certain works sent by authors to some of these. I don't want to say obscure, but let me say less read journals. I have gained some ideas from being a referee. Actually, I have got a collaborator from Iran whom I became aware of by virtue of a paper of his that I had been requested to review for a journal that is published in Iran, and is not one of the big ones. Not like CGASA and The Bulletin of the Iranian Mathematical Society. It's one of the smaller

journals. So, because of refereeing that paper, I saw certain things. It was actually in rings of continuous functions. Just $C(X)$ not $\mathcal{R}L$. I saw some things there that I thought could be considered in $\mathcal{R}L$ and, incidentally, after this fellow's paper had been published (in Topology and its Applications), I wrote to him that I had seen his paper. I didn't tell him that I was the referee. I suggested that he consider working this in Pointfree Topology and he did.

I.N. *How do you feel about rejecting an article as a referee or as the Editor-in-Chief of QM?*

T.D. It's more painful if the rejection is towards somebody I know personally, even if I'm not the one that's rejecting. We send submitted papers to referees and, in particular, I used to avoid handling papers by my colleagues. Some papers by some colleagues of mine have been rejected and I had to communicate that to a colleague. That always gives me a bit of pain. Normally we request two referees at QM. We study their reports and then communicate the decision to the corresponding author.

At QM if we feel that a referee has been somewhat unfair and has written a flimsy report, like for instance: "I do not think that this paper is worthy of publication in a journal of the stature of QM", without going into detail, we send it to another referee for another opinion. At QM the rejection can also be communicated by an associate editor not only by the Editor-in-Chief. All final rejections that I have seen have been based on fair reports.

There was a time when somebody sat with an author's paper for about three months and sent something like: "This paper does not seem to fit in with the current trends of QM". That was so unfair. We had to communicate to the author that there would be a delay because the report that was received required another opinion. For the authors that I don't know personally, I do not feel really that much pain. But for colleagues, I do feel that pain. At times some of them think that *it is you, or it is so and so, it's Dube not wanting to publish my paper.*

I.N. *How do you feel when your own articles are rejected?*

T.D. Regarding the first paper of mine that got rejected, the rejection caused me pain because of the manner in which the referee reported.

It was a paper on nearness spaces. The title and abstract say *nearness* spaces. The introduction says *nearness* spaces. The referee held the paper for about two or three months. He then sent a report that simply said: "This paper makes no contribution towards *uniform* spaces". That made me really very angry. I then spoke to Lamar Bentley (whom I had been introduced to by Ramesh Ori during my PhD studies) and narrated the whole story. Within two or three days Lamar said to me: "Take the paper as is and send it to this particular journal". Incidentally, he was not an editorial board member of that particular journal, I should point that out. I sent the paper there, and it was carefully reviewed. With minor changes the paper, titled *Sigma-compactness via Nearness*, was accepted in Kyungpook Mathematical Journal [4]. That was at the beginning of my career. The original rejection really annoyed me because I did not pretend that the paper was making any contribution to *uniform* spaces, it was about nearness spaces.

I.N *You led the Topology Research Group at UNISA (called TAU - Topology at UNISA) during its inception and you held the Research Chair in Topology within which you steadfastly supported the internationalisation initiatives of the College of Science, Engineering and Technology and the university. You have visited mathematics departments with members of TAU within the Southern African Development Community (SADC) countries notably, Botswana, the Kingdom of Eswatini (the former Swaziland), Mozambique, Namibia, Tanzania, Zambia and Zimbabwe. Some members of TAU have also travelled with you to other African countries on the same initiative including Cameroon, Ghana, Kenya and Rwanda. Can you relate some of your memorable experiences and engagements with students of Africa during these collaborative visits and your influence of pointfree topology research on the continent?*

T.D. First, of all the countries that you have mentioned, I found the students in Cameroon to have been exposed to more mathematics than any of these other African countries. I found that fourth year students knew, for instance, what lattices are – something that's not common in the other African countries that I've been to. I found those students more ready to be brought into my line of thinking because I do

topology in a very lattice theoretic way. It is easy, for me, to work with starting students who already know some bit of lattice theory.

With regard to my influence, let me start about Tanzania. I was with Tega during my first (and second) visit to the University of Dar es Salaam. Both Tega and I gave talks. Tega gave her first talk on her mathematics and another motivational talk trying to motivate students, especially African women to get into mathematics.

There were students who ended up doing their master's degrees mainly by virtue of our visit and us being there, and actually, I ended up being co-supervisor for two of them. Yes - they were registered at the University of Dar es Salaam. One of them is a gentleman called Shegu Mayila and the other a lady called Elizabeth Mrema. They both completed their master's degrees, and Elizabeth is now doing a PhD at the University of Stellenbosch with Sophie Marques (a former postdoctoral fellow of George Janelidze) as her supervisor. Shegu visited our research group and attended a writing retreat, all funded from my grant. Another student, Mr. Almachius Rweju, who's also from Tanzania and who wrote a dissertation on a topic in commutative rings that I had suggested for him has now been accepted to study at the University of Denver.

Now, the policy at the University of Dar es Salaam is that the supervisor must be somebody inside there. The internal person that I have been working with is Dr Marco Mpimbo Kiteu whom I have also once invited to come to South Africa. So the students worked with Dr Mpimbo but the projects were suggested by myself. They were in Topology and in Algebra, commutative rings mainly and I'm very glad that this fellow has been accepted to do a PhD at the University of Denver.

Here is how I met Dorca. In 2016 she wrote to me saying that she would like to study mathematics at UNISA. She's Kenyan. I told her that I was going to be on my second visit to the University of Dar es Salaam. She paid for herself and took a bus (from Kenya) and came to Tega's and my talks in Tanzania. After the talks she said to me: "Definitely, I'm going to do a PhD with you". She had also applied to do a PhD at the University of Pretoria. After having seen the type

of mathematics we do and how we treated her, both Tega and I, she said that she definitely wanted to be like her (meaning Tega). The rest is history.

Now she's got a PhD student (a lady called Grace Wekesho) that she is co-supervising with Tega and one other colleague at her institution in Kenya, the Technical University of Mombasa (TUM). We have a collaboration with her university now. The MOU (Memorandum of Understanding) between UNISA and TUM has been signed. Tega, Dorca and I ensured that the MOU was signed, and it was signed early this year. When Tega and I went to TUM in 2022 we had a meeting with the Vice Chancellor of that university. She's the one who said: "Let's work with you. Let's work with you. Please. I'm going to draft a Memorandum of Understanding. You take it back to South Africa to your university. Let's see if we can work." I'm happy to say now we are running joint master's and doctoral programmes. Grace Wakesho (the student of Tega and Dorca mentioned above) is within this programme, as is another PhD student (a lady called Ann Fwaru) supervised by Dr Michael Munywoki of TUM and me.

One of the students who graduated at TUM (a lady called Sharon Nyokabi) is going to do her master's degree. She's in the process of registering. She has been accepted and I have roped in one of my former students with Tega, who is our colleague, Batsile Tlharesakgosi, to be co-supervisor for Sharon.

Martin Mugochi has got a PhD student at the University of Namibia working on a project that I have proposed. Always when I go to Namibia I give talks and try to encourage students. In Botswana, no student has taken up any studies with us. In Zimbabwe, no nobody in Zim. They tend to do Applied Mathematics in Zim. At AIMS (the African Institute of Mathematical Sciences) Cameroon, yes. I had a student with whom we (Dube, Naidoo, Ghosh and Ighedo) met at AIMS Cameroon – Annette Flavie Ngo Babem. She did her master's degree at UNISA [41] under my supervision. She didn't have to be here (in South Africa), but, as with some of the Tanzanian students, I once funded her to visit our research group and attend one of our writing retreats.

I.N.   *This is really lovely Themba. Cameroon was really the best. We have really enjoyed these visits and the African experiences together with you.*

T.D.   Oh yes. Indeed. And Cameroon was good.

I.N.   *You have a very close relationship with your daughter, Linda. Is there any influence that you have had in her life with mathematics? And is there any fond memories that you can share whilst doing your research in her company?*

T.D.   Yes. My daughter is not a mathematician. The first time I realised that she was never going to be a mathematician and that she did not have the inclination and the way of thinking was when I was trying to tell her about the Theorem of Pythagoras (pictorially) when she was about finishing primary school. I said: *You see, if you take any right angled triangle and you build up a square here and a square there and the square on the hypotenuse, then the square on the hypotenuse is equal to the sum of these two squares.* I remember this vividly. She looked at me and said: "Dad, in *all of them*?" That's when I realised no no, my daughter does not have an inclination for this. But, sadly, no I mean strangely, when it comes to things like writing a little poem or a limerick (you know, limericks like we did at school) she can do it just like that. In English no less, which is her second language. For the life of me, I can't write a limerick. I just don't have the mind for that type of creativity.

I subsequently never used to talk mathematics or results with my daughter. With my niece (Lerato – she's actually my cousin's daughter) I did. There was a time Lerato visited me. She lives in Johannesburg but she was studying at UNISA. She was doing Psychology, or something like that. There was a time I was working on some result and I needed to prove certain sets to cover the space but somehow I could not do that. I could do it contra-positively by showing that the intersection is empty. So, I was busy tormenting myself and that was in locales. You don't work with elements, you know what locales are like. Somehow I had a way of doing it. I was ecstatic. I spoke out aloud. I said: *Lerato, I've finally proved this, man. The intersection is an empty set (I wouldn't tell her it was a void sublocale)! It's empty!,*

I said excitedly. She looked at me and said: "Malume (*that means uncle in our language*), Malume if it's empty is that really exciting?" So I thought to myself: *Ha! here's another one who will never be a mathematician.*

But, with my daughter, no I have never really spoken mathematics to her. She's studied things in the Arts and all that.

I.N. *Themba, this has really been fascinating and how this has turned out. Finally, do you have any parting words to share, to convey to friends, colleagues and students whoever it might be?*

T.D. You know, Inderasan, my students tend to say that I was a good supervisor to them and all that. Well, it's something that I learned from *my* supervisor, Dharms Baboolal. You know, really. You know Dharms used to run lectures on Mondays on frames. Every Monday, I used to attend with Rama Govender, you remember him. Dharms' style of doing things, his way of teaching, he would teach you like you are an entering graduate student. He takes you into his confidence. He motivates you. In terms of the learning of mathematics, the learning of doing research and all that. Apart from being my supervisor, honestly honestly he's been the most and best influence that I have had. And I would like to say to him: *Thank you very much Dharms.* It's not only towards me, he has had other students, Hlengani (Siweya), Simo (Mthethwa), Cerene (Rathilal) and many others. They say exactly the same sort of thing. Dharms has not changed.

One anecdote that I would like to mention about Dharms. Coming from topological spaces, I knew that a bijection is not necessarily a homeomorphism. So one day, I had this one-to-one and onto map in frames. Actually that came up in our lectures. Suddenly he says that it's an isomorphism. I said: *Wait a minute. Dharms, do we not have to check if f^{-1} also preserves the things that need to be preserved?* He said to me: "Check it." That was the first time that somebody at supervisory level did not say: *Okay, let me give you the answer. He said* "Check it" *you know.* That was his way of teaching that some of these things you are not going to find it in this or that particular paper. You can work them out yourself. It's a style that I have imparted towards my students and it's something from which I really

really learnt.

There are times that I have a result that I know in spaces, or maybe it happens in rings. Then what I do with my rings is that I apply a functor to go to algebraic frames. Then I wonder if this has not been done. I quickly check in Martínez' papers, that's where you get your resources for algebraic frames, but I cannot find it. Then I say to myself: *let me check it – as Dharms taught me I should, and boom, here it is.* With my current PhD student, Mbekezeli Nxumalo, he once asked of me: "Is it the case that every sublocale of a Boolean frame is Boolean?" I said: *Okay Mbekezeli, check it. What needs to be checked?* He said to me: "Yeah Prof, I did check it. I took this sublocale but I still couldn't work out if it's Boolean." I then said to him: *Sublocales and homomorphic images, what do you know about them?* He then realised: "Ah, I can check homomorphic images. I know that lattice homomorphisms preserve complemented elements. There it is." Because of the way he is, he is very very sharp, but he was trying to do it the hard way.

It's a style of imparting knowledge to young ones that I inherited from my supervisor. Dharms would not always just give you the answer and furthermore, Dharms would never try to pull wool over your eyes – maybe that happens with all mathematicians. If the answer is not immediately there he would tell you that he never looked at it and maybe we should look into it. But, if it's something that he knows and that he believes you can be able to work out, he'll tell you as a student: "Check it".

With regard to my students, I would say also: *Ask questions, even if you think it's a dumb question. Please ask questions.* Out of the questions that you ask, things come up. Inderasan, you know, we would ask each other questions in a train in California and at the airport, you remember in Portugal. I would put a question to you and you would do the same to me and boom, we would invariably have a paper emanating out of that. So to my students: *Ask questions.*

I.N. *Themba this has been truly remarkable and a wonderful opportunity. I can also relate about Dharms. He was my first year undergraduate Algebra lecturer at UDW and ended up being my master's supervisor. I*

should point out Ramesh Ori, as well, who was also my co-supervisor. Dharms has not changed indeed. Also, a Thank you to Dharms from me. Themba, thank you so very much for your precious time and the very best wishes to you for longevity into your retirement. □

3 Epilogue

The historical account of the research that emanated and the many mathematical relationships that Dube shared with distinguished scholars, colleagues and students that's narrated in the interview certainly contributes importantly and significantly to the history within the South African mathematical landscape, to the African mathematical diaspora and to that within the global pointfree topology community. Dube's contributions are unique in many ways and will be infinitely admired and remembered.

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