



Categories and General Algebraic Structures with Applications

Volume 17, Number 1, July 2022
ISSN Print: 2345-5853 Online: 2345-5861



Shahid Beheshti University
<http://cgasa.sbu.ac.ir>

In the Name of God

Categories and General Algebraic Structures with Applications

Editor-in-Chief:
M.Mehdi Ebrahimi
m-ebrahimi@sbu.ac.ir

Director in Charge:
Mojgan Mahmoudi
m-mahmoudi@sbu.ac.ir

Manager:
Meysam Madani
me@madani.pro

Editorial Board:

M.Mehdi Ebrahimi, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran, *m-ebrahimi@sbu.ac.ir* (*Editor-in-Chief*).

Reza Ameri, School of Mathematics, Statistics, and Computer Science, University of Tehran, Tehran, Iran, *rameri@ut.ac.ir*.

Fariborz Azarpanah, Department of Mathematics, Shahid Chamran University, Ahvaz, Iran, *azarpanah@ipm.ir*.

Rajabali Borzooei, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran, *borzooei@sbu.ac.ir*.

Amir Daneshgar, Department of Mathematical Sciences, Sharif University of Technology, Tehran, Iran, *daneshgar@sharif.ir*.

Themba Dube, Department of Mathematical Sciences, School of Science, University of South Africa, Johannesburg, South Africa, *dubeta@unisa.ac.za*.

Ali Akbar Estaji, Department of Pure Mathematics, School of Mathematical Sciences, Hakim Sabzevari University, Sabzevar, Iran, *aaestaji@hsu.ac.ir*.

Akbar Golchin, Department of Mathematics, School of Mathematics and Computer Science, Sistan Baluchestan University, Zahedan, Iran, *agdm@hamoon.usb.ac.ir*.

Victoria Gould, Department of Mathematics, Faculty of Sciences, University of York, York, United Kingdom, *varg1@york.ac.uk*.

Naser Hosseini, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Bahonar University, Kerman, Iran, *nhosseini@mail.uk.ac.ir*.

Ali Madanshekaf, Department of Mathematics, Faculty of Mathematical Sciences, Semnan University, Semnan, Iran, *amadanshekaf@semnan.ac.ir*.

Mojgan Mahmoudi, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran, *m-mahmoudi@sbu.ac.ir* (*Director in Charge*).

Mohammad Reza Radjatzadeh Moghadam, Department of Pure Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran, *moghadam@math.um.ac.ir*.

Morteza Moniri, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran, *m-moniri@sbu.ac.ir*.

Alireza Salemkar, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran, *salemkar@sbu.ac.ir*.

Honorary Associate Editors:

Bernhard Banaschewski, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, L8S 4K1, Canada.

Kiran R. Bhutani, Department of Mathematics, The Catholic University of America, Washington, D.C. 20064, United States of America, *bhutani@cua.edu*.

Guillaume Brummer, Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch, 7701, South Africa, *gcl.brummer@uct.ac.za*.

Sydney Bulman-Fleming, Department of Mathematics, Wilfrid Laurier University, 75 University Avenue West, Waterloo, Ontario N2L 3C5, Canada, *sbulman@wlu.ca*.

Maria Manuel Clementino, Department of Mathematics, University of Coimbra, Apartado 3008, 3001-454, Coimbra, Portugal, *mmc@mat.uc.pt*.

Piergiulio Corsini, Department of Civil Engineering and Architecture, Via delle Scienze 208, 33100 Udine, Italy, *piergulio.corsini@uniud.it*.

Brian A. Davey, Department of Mathematics and Statistics, La Trobe University, Bundoora (Melbourne) VIC 3083, Australia, *b.davey@latrobe.edu.au*.

Dikran Dikranjan, Department of Mathematics and Computer Science, University of Udine, Via delle Scienze, 208 – Loc. Rizzi, 33100 Udine, Italy, *dikranjan@dimi.uniud.it*.

Abbas Edalat, Department of Computing, Imperial College London, 180 Queen's Gate, London SW7 2RH, United Kingdom, *a.edalat@imperial.ac.uk*.

Majid Ershad, Department of Mathematics, Shiraz University, Shiraz, Iran, *ershad@shirazu.ac.ir*.

Christopher Gilmour, Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch, 7701, South Africa, *christopher.gilmour@uct.ac.za*.

Eraldo Giuli, Dipartimento di Matematica Pura ed Applicata, Università di L'Aquila, Coppito 67010, L'Aquila, Italy, *giuli@univaq.it*.

Rob Goldblatt, School of Mathematics and Statistics, Victoria University of Wellington, Wellington 6140, New Zealand, *rob.goldblatt@msor.vuw.ac.nz*.

Marino Gran, Institut de Recherche en Mathématique et Physique Chemin du Cyclotron 2, Université Catholique de Louvain, 1348 Louvain-la-Neuve, Belgium, *marino.gran@uclouvain.be*.

Marco Grandis, Dipartimento di Matematica, Università di Genova, Via Dodecaneso 35, 16146 Genova, Italy, *grandis@dimi.unige.it*.

Hossein Hajiabolhassan, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Evin 19839, Tehran, Iran, *hhaji@sbu.ac.ir*.

Wei He, School of Mathematical Sciences, Nanjing Normal University, China, *weihe@njnu.edu.cn*.

Dirk Hofmann, Departamento de Matemática, Universidade de Aveiro, 3810-193 Aveiro, Portugal, *dirk@ua.pt*.

Martin Hyland, Department of Pure Mathematics and Mathematical Statistics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WB, United Kingdom, *m.hyland@dpmms.cam.ac.uk*.

Ali Iranmanesh, Department of Mathematics, Tarbiat Modares University, P.O. Box 14115-137, Tehran, Iran, *iranmana@modares.ac.ir*.

George Janelidze, Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch, 7701, South Africa, *janelg@telkomsa.net*.

Peter T. Johnstone, Department of Pure Mathematics and Mathematical Statistics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WB, United Kingdom, *p.t.johnstone@dpmms.cam.ac.uk*.

Vinayak Joshi, Department of Mathematics, University of Pune, Ganeshkhind, Pune, Maharashtra 411007, India, *vvj@math.unipune.ac.in*.

André Joyal, Département de Mathématiques, Université du Québec à Montréal, C.P. 8888, Succ. Centre-ville Montréal, QC H3C 3P8, Canada, *joyal.andre@uqam.ca*.

Achim Jung, School of Computer Science, The University of Birmingham, Edgbaston, Birmingham, B15 2TT, United Kingdom, *a.jung@cs.bham.ac.uk*.

Bakhadyr Khoussainov, Department of Computer Science, University of Auckland, Private Bag 92019, Auckland, New Zealand, *bmk@cs.auckland.ac.nz*.

Mati Kilp, Institute of Pure Mathematics, University of Tartu, 50409 Tartu, Estonia, *mkilp@math.ut.ee*.

Ulrich Knauer, Institut für Mathematik, Carl von Ossietzky Universität Oldenburg, D-26111 Oldenburg, Germany, *knauer@mathematik.uni-oldenburg.de*.

Hans-Peter Kunzi, Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch, 7701, South Africa, *hans-peter.kunzi@uct.ac.za*.

Valdis Laan, Institute of Pure Mathematics, University of Tartu, 50409 Tartu, Estonia, *vlaan@ut.ee*.

F. William Lawvere, Department of Mathematics, University at Buffalo, 244 Mathematics Building, Buffalo, New York 14260-2900, United States of America, *wlawvere@buffalo.edu*.

Jorge Martinez, Department of Mathematics, University of Florida, Gainesville, FL 32611-8105, United States of America, *martinbad@ufl.edu*.

M. Andrew Moshier, Department of Mathematics and Computation, Faculty of Mathematics, Physics and Computation, Schmid College of Science and Technology, Chapman University, Orange, CA 92866, United State of America, *moshier@chapman.edu*.

Peeter Normak, Institute of Informatics, Tallinn University, Narva Road 25, Tallinn 10120, Estonia, *peeter.normak@tlu.ee*.

Robert Paré, Department of Mathematics and Statistics, Dalhousie University, 6316 Coburg Road, Halifax, Nova Scotia, Canada B3H 3J5, *pare@mathstat.dal.ca*.

Jorge Picado, Department of Mathematics, University of Coimbra, Apartado 3008, 3001-454, Coimbra, Portugal, picado@mat.uc.pt.

Andrew Pitts, Computer Laboratory, University of Cambridge, William Gates Building, 15 JJ Thomson Avenue, Cambridge CB3 0FD, United Kingdom, andrew.pitts@cl.cam.ac.uk.

Masoud Pourmahdian, Department of Mathematics and Computer Sciences, Amirkabir University of Technology, Tehran, Iran; School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5746, Tehran, Iran, pourmahd@ipm.ir.

Jim Renshaw, School of Mathematics, University of Southampton, Highfield, Southampton, SO17 1BJ, United Kingdom, j.h.renshaw@maths.soton.ac.uk.

Jan Rutten, Faculty of Science, Radboud University, Comeniuslaan 4, 6525 HP Nijmegen, The Netherlands, jan.rutten@cwi.nl.

Hanamantagouda P. Sankappanavar, Department of Mathematics, State University of New York, New Paltz, New York 12561, United States of America, sankapph@hawkmil.newpaltz.edu.

Gavin J. Seal, Mathematics Section, Ecole Polytechnique Federale de Lausanne (EPFL), GH-1015 Lausanne, Switzerland, gavin.seal@epfl.ch.

Omid Ali Shahni Karamzadeh, Department of Mathematics, Shahid Chamran University, Ahvaz, Iran, karamzadeh@cua.ac.ir.

Kunitaka Shoji, Department of Mathematics, Shimane University, Matsue, Shima 690, Japan, ksho@riko.shimane-u.ac.jp.

Manuela Sobral, Department of Mathematics, University of Coimbra, Apartado 3008, 3001-454, Coimbra, Portugal, sobral@mat.uc.pt.

Andrzej Tarlecki, Faculty of Mathematics, Informatics, and Mechanics, Institute of Informatics, University of Warsaw, Banacha 2, 02-097 Warszawa, Poland, a.tarlecki@mimuw.edu.pl.

Walter Tholen, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada, tholen@mathstat.yorku.ca.

Aims and Scope:

“**Categories and General Algebraic Structures with Applications**” is an international journal published by Shahid Beheshti University, Tehran, Iran, founded in 2013. It is **open access**, also it is free of any publication charges. It publishes original high quality research papers and invited research and survey articles mainly in two subjects: **Categories** (algebraic, topological, and applications in mathematics and computer science) and **General Algebraic Structures** (not necessarily classical algebraic structures, but universal algebras such as algebras in categories, semigroups, their actions, automata, ordered algebraic structures, lattices (of any kind), quasigroups, hyper universal algebras), and their applications.

Submission:

Your manuscripts should be written in good English using LaTeX. By submitting, the authors confirm that their articles have been neither published nor submitted for publication elsewhere. Submissions should be made online in the web site and by emailing the PDF file of your manuscripts to the *Editor-in-Chief* at: m-ebrahimi@sbu.ac.ir.

Refereeing Process:

Our review process type is the single-blind peer review.

After the Editorial Board approves that a submitted paper meets the aims and scope of the journal, it will be sent to one or two experts, advised by the Editorial Board or by a member of the Honorary Associate Editors, to review it. The refereeing should be completed in three months, unless the paper is longer than 15 pages. Based on the referee’s report, the Editorial Board decides on the paper as:

The paper is accepted without changes; The paper is accepted subject to minor changes, in which case the revised version should be sent back to the journal in at most one month; The paper will be accepted subject to more substantial changes being made, in which case the revised version should be sent back to the journal in at most two months; The paper might be acceptable but requires major revisions, in which case the paper will be regarded as withdrawn and any new version will be regarded as a new submission; The paper is rejected.

Accepted Articles:

If a paper is accepted, the corresponding author will be required to provide the LaTeX file of it, **without changes**, using the style of the journal given in the web page: <http://cgasa.sbu.ac.ir>. Please ensure the **References** are produced in the same style as the sample article, also notice that we apply Mathematical Reviews abbreviations list for the journal titles. Accepted papers should also have a **Persian abstract**, which can be prepared with the help of the editorial board at m-mahmoudi@sbu.ac.ir.

Copyright:

The authors of the accepted papers, by sending the final original files of their articles, agree that the copyright of their articles are transferred to this journal. Also, they should notice that we follow the CC BY creative commons copyright license.

Publication Ethics:

We agree to and follow all ethical behavior for publishing, all about authors, editors, peer reviewers, and peer reviewing process. We follow the COPE publication ethics.

Indexing Databases:

ESCI, Web of Science



Elsevier's Scopus



ISC, Islamic Science Citation Index



Scientific-Research degree by the Iranian Ministry of Science, Research, and Technology



Mathematical Reviews (MR)



Zentralblatt Math



Journal Address:

Categories and General Algebraic Structures with Applications, Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Evin 19839, Tehran, Iran. Telefax: 0098 21 22431652, Web page: <http://cgasa.sbu.ac.ir>.

Subscription: Please contact the Editorial Office in the web site for subscription information.

Publisher: Shahid Beheshti University.

Contents

K. Roberto and H. Mariano K-theories and free inductive graded rings in abstract quadratic forms theories	1
A.P.K. Craig, B.A. Davey, and M. Haviar Expanding Belnap 2: the dual category in depth	47
Z. Keshtkar, R. Mohamadian, M. Namdari, and M. Zeinali On some properties of the space of minimal prime ideals of $C_c(X)$	85
P. Lipparini Universal extensions of specialization semilattices	101
G. Aytekin Arıncı and T. Şahan Coverings and liftings of generalized crossed modules	117
A. Hosseinabadi, M. Haddadi, and Kh. Keshvardoost On nominal sets with support-preorder	141
H. Tiwari and R. Srivastava On injective objects and existence of injective hulls in $\mathcal{Q}\text{-TOP}/(Y, \sigma)$	173
H. Choulli, K. Draoui, H. Mouanis Quantum determinants in ribbon category	203

