

# Categories and General Algebraic Structures with Applications

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## Categories and General Algebraic Structures with Applications

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