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Reflectional topology in MV-algebras

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Abstract. In this paper, we define soaker ideals in an MV-algebra, and study the relationships between soaker ideals and the other ideals in an involutive MV-algebras. Then we introduce a topology on the set of all the soaker ideals, which we call reflectional topology, and give a basis for it. By defining the notion of join-soaker ideals, we show that the reflectional topology is compact. We also give a characterization of connectedness of the reflectional topology. Finally, we investigate the properties of T_0 and T_1-space in this topology.

1 Introduction and preliminaries

MV-algebras have been introduced by C.C. Chang in 1958 [1] to give an algebraic counterpart of the multiple-valued Łukasiewicz propositional logic.

After their introduction by Chang, MV-algebras free themselves from the bonds of logic and become an autonomous mathematical discipline with deep connections to several other branches of mathematics. For example, in 1986 D. Mundici proved that the category of lattice ordered abelian groups

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with strong unit is categorically equivalent to the category of MV-algebras ([9]). This result is very important because lattice ordered abelian groups do not set up an equational variety unlike MV-algebras. In this way more complicated properties in the group theory language can became simpler in the language of MV-algebras. Moreover, the study of normal forms for Lukasiewicz logic brought to a deep relation between MV-algebras and toric varieties through the concept of Schauder bases, which are the affine versions of a complex of nonsingular cones.

We recall that if A is an $MV-$algebra, then we denote by $Spec(A)$ the set of prime ideals of A and $Spec(A)$ can be endowed with the spectral topology. The topological space $Spec(A)$ is called the *prime spectrum* of A. Recently, F. Forouzesh et al. introduced the inverse topology on $Min(A)$ and proved that it is compact, Hausdorff, T_0, and T_1 [6].

In this paper, we recall some facts concerning MV-algebras, introduce soaker ideals in an MV-algebra A, investigate some relationships between the soaker ideals and the other ideals of an involutive MV-algebra and give a characterization of soaker ideals and study some properties of them.

Then on the set $X = Refl(A)$, of soaker ideals, we define a topology, τ_X, called the reflectional topology. We show it is an Alexandrov topology and give a basis for it. We define join-soaker ideals in MV-algebras and give several characterizations for it. We give a condition under which a certain subset of X is compact; and we show that if A is a join-soaker ideal, then X is compact. Also, we give a characterization of clopen sets and use that to characterize connectedness of the reflectional topology τ_X. We also prove for a Boolean algebra A, the topology is disconnected for a non-trivial X. Finally, we show that the reflectional topology is T_0; and we give necessary and sufficient conditions for X to be T_1. We show Hausdorffness is equivalent to being T_1.

In all the sections, we recollect some definitions and results which will be used in what follows.

Definition 1.1. [1] An MV-algebra is a structure $(A, \oplus, {}^*, 0)$, where $\oplus$ is a binary operation, *, is a unary operation, and 0 is a constant such that the following axioms are satisfied for any $a, b \in A$:
$(MV1)$ $(A, \oplus, 0)$ is an abelian monoid,
$(MV2)$ $(a^*)^* = a$,
$(MV3)$ $0^* \oplus a = 0^*$,

$(MV4)$ $(a^* \oplus b)^* \oplus b = (b^* \oplus a)^* \oplus a$.

Note that $1 = 0^*$ and the auxiliary operation $\odot$ is defined as follows:

$$x \odot y = (x^* \oplus y^*)^*.$$

Lemma 1.2. [10] *Let A be an MV-algebra. For $x, y \in A$, the following conditions are equivalent:*

(1) $x^* \oplus y = 1$,
(2) $x \odot y^* = 0$,
(3) *There is an element $z \in M$ such that $x \oplus z = y$,*
(4) $y = x \oplus (y \ominus x)$.
For any two elements $x, y \in A$, $x \leq y$ iff x and y satisfy the equivalent conditions (1)-(4)*, where $\leq$ is the natural order of A.*

We recall that the natural order determines a bounded distributive lattice structure such that

$$x \vee y = x \oplus (x^* \odot y) = y \oplus (x \odot y^*) \quad \text{and} \quad x \wedge y = x \odot (x^* \oplus y) = y \odot (y^* \oplus x).$$

Lemma 1.3. [10] *In each MV-algebra A, the following hold for all $x, y, z \in A$:*

(1) $x \leq y$ *if and only if* $y^* \leq x^*$,
(2) *If $x \leq y$, then $x \oplus z \leq y \oplus z$ and $x \odot z \leq y \odot z, x \wedge z \leqslant y \wedge z$,*
(3) $x \leq y$ *if and only if* $x^* \oplus y = 1$ *if and if* $x \odot y^* = 0$,
(4) $x, y \leq x \oplus y$ *and* $x \odot y \leq x, y$, $x \leq nx = x \oplus x \oplus \cdots \oplus x$ *and* $x^n = x \odot x \odot \cdots \odot x \leq x$,
(5) $x \oplus x^* = 1$ *and* $x \odot x^* = 0$,
(6) *If $x \leq y$ and $z \leq t$, then $x \oplus z \leq y \oplus t$,*
(7) $x \odot (y \vee z) = (x \odot y) \vee (x \odot z)$,
(8)$x \wedge (y \oplus z) \leqslant (x \wedge y) \oplus (x \wedge z)$, *hence* $x \wedge (x_1 \oplus \cdots \oplus x_n) \leqslant (x \wedge x_1) \oplus \cdots \oplus (x \wedge x_n)$, *for all* $x_1, \ldots, x_n \in A$.

Definition 1.4. [2] An ideal of an MV-algebra A is a nonempty subset I of A satisfying the following conditions:

$(I1)$ If $x \in I$, $y \in A$ and $y \leq x$ then $y \in I$,
$(I2)$ If $x, y \in I$, then $x \oplus y \in I$.
We denote by $Id(A)$ the set of ideals of an MV-algebra A.

Definition 1.5. [2] Let P be an ideal of an MV-algebra A. Then P is a proper ideal of A if $P \neq A$. An ideal I of an MV-algebra A is called

- [2] prime ideal if for all $x, y \in A, x \wedge y \in P$ yields $x \in P$ or $y \in P$.
- [10] finitely meet-irreducible if $I \cap J \subseteq P$, then $I \subseteq P$ or $J \subseteq P$, for all $I, J \in Id(A)$.

Lemma 1.6. [10] *In an MV-algebra A, the ideal P is finitely meet-irreducible if and only if P is prime ideal of A.*

Definition 1.7. [2, 8] Let X and Y be two MV-algebras. A function $f : X \to Y$ is called a *homomorphism* of MV-algebras if and only if

(1) $f(0) = 0$,
(2) $f(x \oplus y) = f(x) \oplus f(y)$,
(3) $f(x^*) = (f(x))^*$.

Remark 1.8. [2] We recall that for a nonempty subset $X \subseteq A$, the smallest ideal of A which contains X, that is, $\bigcap\{I \in Id(A) : X \subseteq I\}$, is said to be the ideal of A generated by X and will be denoted by $(X]$.

Remark 1.9. [10] Let $X \subseteq A$. We have

(1) $(X] = \{a \in A : a \leqslant x_1 \oplus x_2 \oplus ... \oplus x_n,$ *for some* $n \in \mathbb{N}$ *and* $x_1, ..., x_n \in X\}$. $I \in Id(A)$ is called a finitely generated ideal, if $I = (x_1, ..., x_n]$, *for some* $x_1, x_2, ..., x_n \in A$ *and* $n \in \mathbb{N}$. In particular, $(a] = \{x \in A : x \leqslant na,$ *for some* $n \in \mathbb{N}\}$.
(2) *For* $I_1, I_2 \in Id(A), I_1 \vee I_2 = \{x \in A : \exists a_i \in I; x \leqslant a_1 \oplus a_2\}$.
(3) $(a] \wedge (b] = (a \wedge b]$.
(4) if $a \leq b$, then $(a] \subseteq (b]$.

Proposition 1.10. [10] *Let $f : A \to B$ be a homomorphism of MV-algebras.*

(i) *if J is a proper ideal of B, then $f^{-1}(J)$ is a proper ideal of A.*
(ii) *if f is onto and I is an ideal of A, then $f(I)$ is an ideal of B.*

Lemma 1.11. [10] *Let $f : A \to B$ be an onto homomorphism of MV-algebras A, B and $\bigvee_{i \in I} I_i$ be a family of ideals of A. Then we have $f(\bigvee_{i \in I} I_i) = \bigvee_{i \in I} f(I_i)$.*

Definition 1.12. [2] Let X be a nonempty subset of MV-algebra A and $Ann_A(X)$ be the annihilator of X defined by

$$Ann_A(X) = \{a \in A : a \wedge x = 0, \forall x \in X\}.$$

2 Soaker ideals in MV-algebras

In the sequel, A is an MV-algebra.

Definition 2.1. Let I be an ideal of A. I is called a *soaker ideal* of A, if

(i) $I \neq \{0\}$

(ii) For $\{I_i\}_{i\in J} \subseteq Id(A)$, If $I \subseteq \bigvee_{i\in J} I_i$, then $I \subseteq I_i$, for some $i \in J$.

Example 2.2. Let $A = \{0, a, b, 1\}$, where $0 < a, b < 1$. Define $\odot$, $\oplus$ and $*$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

$\oplus$	0	a	b	1
0	0	a	b	1
a	a	a	1	1
b	b	1	b	1
1	1	1	1	1

$*$	0	a	b	1
	1	b	a	0

Then $(A, \oplus, \odot, *, 0, 1)$ is an MV-algebra [8]. It is clear that $I_1 = \{0\}$, $I_2 = \{0, a\}$, $I_3 = \{0, b\}$ and $I_4 = A$ are ideals of A and I_2 and I_3 are soaker ideals, while A is not a soaker ideal because $A \subseteq I_2 \vee I_3$. But $A \not\subseteq I_2$ and $A \not\subseteq I_3$.

Example 2.3. Consider $S_1 = \{0, 1\}$, $S_2 = \{0, 1/2, 1\}$. Then $A = S_1 \times S_2$ with operations $(x, y) \oplus (z, t) = (min\{1, x + z\}, min\{1, y + t\})$ and $(x, y)^* = (1 - x, 1 - y)$ is an MV-algebra. We have $I_0 = \{(0, 0)\}$, $I_1 = \{(0, 0), (1, 0)\}$, $I_2 = \{(0, 0), (0, 1/2), (0, 1)\}$ and A are ideals of A. It can be easily verified that I_1, I_2 are soaker ideals, while A is not a soaker ideal.

Example 2.4. Let $A = \{0, a, b, c, d, e, f, 1\}$ is rectangular cube such that $0 < a, d < e < 1$, $0 < a, b < c < 1$, $0 < b, d < f < 1$, $0 < d < e, f < 1$, $0 < a < c, e < 1$ and $0 < b < c, f < 1$.

Define $\oplus$ and $*$ as follows:

$\oplus$	0	a	b	c	d	e	f	1
0	0	a	b	c	d	e	f	1
a	a	a	c	c	e	e	1	1
b	b	c	b	c	f	1	f	1
c	c	c	c	c	1	1	1	1
d	d	e	f	1	d	e	f	1
e	e	e	1	1	e	e	1	1
f	f	1	f	1	f	1	f	1
1	1	1	1	1	1	1	1	1

$*$	0	a	b	c	d	e	f	1
	1	f	e	d	c	b	a	0

Then $(A, \oplus, *, 0, 1)$ is an MV-algebra [8], it is clear that $I_0 = \{0\}$, $I_1 = \{0, a\}$, $I_2 = \{0, d\}$, $I_3 = \{0, b\}$, $I_4 = \{0, a, d, e\}$, $I_5 = \{0, a, b, c\}$, $I_6 = \{0, b, d, f\}$ and $I_7 = A$ are ideals of A. We can easily see that I_1, I_2 and I_3 are soaker ideals of A,but I_4, I_5, I_6 and I_7 are not soaker ideals. For example, $I_4 \subseteq I_1 \vee I_2$. But $I_4 \nsubseteq I_1$ and $I_4 \nsubseteq I_2$. Hence I_4 is not a soaker ideal of A.

Example 2.5. Let $G = \oplus\{Z_i/i \in \mathbb{N}\}$ be the lexicographic product of denumerable infinite copies of the abelian l-group $\mathbb{Z}$ of the relative integers and $e^i \in G$ such that $e^i_k = 0$ if $k \neq i$ and $e^i_k = 1$ if $k = i$.
Consider the perfect MV-algebra $A = \Gamma(G)$, where Γ is a functor from the category of abelian l-groups to the category of prefect MV-algebras [5].
If we set $P_i =< (0, e^i) >$, then $P_i \subseteq P_j$, for $i > j$, and hence the set of all soaker ideals of A is $\{P_i/i \in \mathbb{N}\}$.

Theorem 2.6. *S is a soaker ideal of A if and only if $S = \bigvee_{i \in J} I_i$ implies $S = I_i$, for some $i \in J$.*

Proof. Let S be a soaker ideal and $S = \bigvee_{i \in J} I_i$. Since $S \subseteq \bigvee_{i \in I} I_i$, we get $S \subseteq I_i \subseteq \bigvee_{i \in J} I_i = S$. Hence $S = I_i$.
Conversely, suppose that $S \subseteq \bigvee_{i \in J} I_i$. Then $S \cap \bigvee_{i \in J} I_i = \bigvee_{i \in J}(S \cap I_i) = S$, by hypothesis, $S \cap I_i = S$, for some $i \in J$. Thus $S \subseteq I_i$, for some $i \in J$. Therefore S is a soaker ideal of A. $\square$

Lemma 2.7. *In an MV-algebra A, we have* $(a] \vee (b] = (a \vee b] = (a \oplus b]$.

Proof. Since $a, b \leq a \vee b \leq a \oplus b$, by Remark 1.9(4), we deduce that $(a], (b] \subseteq (a \vee b] \subseteq (a \oplus b]$. Hence $(a] \vee (b] \subseteq (a \vee b] \subseteq (a \oplus b]$.
Conversely, let $x \in (a \oplus b]$. It follows from Remark 1.9 (1), (2) that for some $n \geq 1$,

$$x \leq n(a \oplus b) = na \oplus nb \in (a] \vee (b].$$

Hence $x \in (a] \vee (b]$. That is, $(a \oplus b] \subseteq (a] \vee (b]$. Thus we have $(a] \vee (b] = (a \oplus b]$. So $(a] \vee (b] = (a \vee b] = (a \oplus b]$. □

Since for any ideal I, $I = \bigvee_{a \in I} (a]$, we have:

Corollary 2.8. *Every soaker ideal is principal.*

Setting $C = \{z \in A : \forall x, y \in A, z \leq x \oplus y \text{ implies } \exists m \in \mathbb{N}, z \leq mx \text{ or } \exists n \in \mathbb{N}, z \leq ny\}$, we have:

Theorem 2.9. $(a]$ *is soaker if and only if* $a \in C$.

Proof. Suppose that $(a]$ is soaker. If $a \leq x \oplus y$, then by Remark 1.9 (4), (3), we get $(a] \subseteq (x] \vee (y]$. Hence $(a] \subseteq (x]$ or $(a] \subseteq (y]$. So $a \leq mx$, for some $m \in \mathbb{N}$ or $a \leq ny$, for some $n \in \mathbb{N}$. Thus $a \in C$.

Now, suppose $a \in C$. If $(a] \subseteq \vee_{\alpha \in I} (a_\alpha]$, then $a \in \bigvee_{\alpha \in I} (a_\alpha]$. We obtain $a \leq a_{\alpha_1} \oplus \cdots \oplus a_{\alpha_k}$. By hypothesis with some manipulations one obtains, $a \leq m_1 a_{\alpha_1}$ or $a \leq m_2 a_{\alpha_2}$ or $\cdots$ or $a \leq m_k a_{\alpha_k}$. We conclude that $(a] \subseteq (a_{\alpha_1}]$ or $(a] \subseteq (a_{\alpha_2}]$ or $\cdots$ or $(a] \subseteq (a_{\alpha_k}]$. Thus $(a]$ is soaker. □

Lemma 2.10. *If* $a \in C$*, then for all* $n \in \mathbb{N}$*,* $na \in C$*; and if there exists* $n \in \mathbb{N}$ *such that* $na \in C$*, then* $a \in C$

Proof. Suppose that $a \in C$. Let $n \in \mathbb{N}$ and $na \leq x \oplus y$. Since $a \leq na$, we get $a \leq x \oplus y$. So $a \leq kx$ or $a \leq ly$. Thus $na \leq knx$ or $na \leq lny$. Therefore $na \in C$.

Now, suppose there exists $n \in \mathbb{N}$ such that $na \in C$. If $a \leq x \oplus y$, then $na \leq nx \oplus ny$. By hypothesis, we have $na \leq (nk)x$ or $na \leq (nl)y$. Since $a \leq na$, we deduce that $a \leq (nk)x$ or $a \leq (nl)y$. Therefore $a \in C$. □

Theorem 2.11. *Let* $f : A \to B$ *be a homomorphism of MV-algebras and* S *be a soaker ideal of* B*. Then* $f^{-1}(S)$ *is a soaker ideal of* A*.*

Proof. Clearly, $f^{-1}(S)$ is an ideal of A. If $f^{-1}(S) = \{0\}$, then $f(\{0\}) = S$, and so $\{0\} = S$, which is a contradiction. Hence $f^{-1}(S) \neq \{0\}$.
Let $\{I_i\}_{i\in J}$ be a family of ideals of A such that $f^{-1}(S) \subseteq \bigvee_{i\in J} I_i$. It follows from Lemma 1.11 that $S \subseteq f(\bigvee_{i\in J} I_i) = \bigvee_{i\in J} f(I_i)$. Now, since S is a soaker ideal of B, we get $S \subseteq f(I_i)$, for some $i \in J$. Hence $f^{-1}(S) \subseteq I_i$, for some $i \in J$. Thus $f^{-1}(S)$ is a soaker ideal of A. $\square$

Theorem 2.12. *Let S be a soaker ideal of A and $f : A \to B$ be an isomorphism of MV-algebras. Then $f(S)$ is a soaker ideal of B.*

Proof. Let $\{I_i\}_{i\in J}$ be a family of ideals of B such that $f(S) \subseteq \bigvee_{i\in J} I_i$. Since f is onto, we get $I_i = f(I'_i)$, for some $I'_i \in Id(A)$, and so $f(S) \subseteq \bigvee_{i\in I} f(I'_i)$, for some $I'_i \in Id(A)$. By Lemma 1.11, we have $f(S) \subseteq f(\bigvee_{i\in J} I'_i)$ and since f is one to one, so $S \subseteq \bigvee_{i\in J} Ii'$. By hypothesis, $S \subseteq I'_i$, for some $i \in J$. Hence $f(S) \subseteq f(I'_i) = I_i$, for some $i \in J$. Thus, $f(S)$ is a soaker ideal of B. $\square$

Definition 2.13. An MV-algebra A is called involutive if $I = Ann(Ann(I))$, for every ideal I of A.

Example 2.14. (i) MV-algebra in Example 2.2 is involutive.
(ii) MV-algebra in Example 2.5 is involutive.

Example 2.15. Let $C = \{0, c, 2c, 3c, \ldots, 1-2c, 1-c, 1\}$ be the MV-algebra defined in [1] with operations as follows:
if $x = nc$ and $y = mc$, then $x \oplus y := (m + n)c$,
if $x = 1 - nc$ and $y = 1 - mc$, then $x \oplus y := 1$,
if $x = nc$ and $y = 1 - mc$ and $m \leq n$, then $x \oplus y := 1$,
if $x = nc$ and $y = 1 - mc$ and $n < m$, then $x \oplus y := 1 - (m - n)c$,
if $x = 1 - mc$ and $y = nc$ and $m \leq n$, then $x \oplus y := 1$,
if $x = 1 - mc$ and $y = nc$ and $n < m$, then $x \oplus y := 1 - (m - n)c$,
if $x = nc$, then $x^* := 1 - nc$,
if $x = 1 - nc$, then $x^* := nc$.

We see that C is a prefect MV-algebra (that is, C has only a maximal ideal) and it has only three ideals: $\{0\}$, $M = \{0, c, 2c, 3c, \ldots\}$ and C. Since $Ann(Ann(M)) = Ann(\{0\}) = A \neq M$, C is not involutive.

Lemma 2.16. *Let A be an MV-algebra and $I, J \in Id(A)$. We have*

(i) $I \subseteq Ann(Ann(I))$,

(ii) *if* $I \subseteq J$, *then* $Ann(J) \subseteq Ann(I)$,

(iii) $Ann(I \cap J) = Ann(I) \cap Ann(J)$,

(iv) $Ann(I \vee J) \supseteq Ann(I) \cap Ann(J)$.

Proof. (i) Let $a \in I$. Suppose that $x \in Ann(I)$. Hence $x \wedge t = 0$, for all $t \in I$. Since $a \in I$, it follows that $x \wedge a = 0$, for $x \in Ann(I)$. Thus $a \in Ann(Ann(I))$. Therefore we get $I \subseteq Ann(Ann(I))$.

(ii) Suppose that $a \in Ann(J)$. Hence $a \wedge x = 0$, for all $x \in J$. We get $a \wedge x = 0$, for all $x \in I$. Then we obtain $a \in Ann(I)$. Thus $Ann(J) \subseteq Ann(I)$.

(iii) We have $a \in Ann(I \cap J)$ if and only if $a \wedge x = 0$, for all $x \in I \cap J$ if and only if $a \wedge x = 0$, for all $x \in I$ and $x \in J$ if and only if $a \in Ann(I)$ and $a \in Ann(J)$ if and only if $a \in Ann(I) \cap Ann(J)$.

(iv) Suppose that $a \in Ann(I) \cap Ann(J)$. Hence $a \wedge t = 0$ and $a \wedge s = 0$, for all $t \in I$ and $s \in J$.

Now, for all $x \in I \vee J$, by Remark 1.9 (2), we get $x \leq c \oplus b$, for some $c \in I$ and $b \in J$. It follows from Lemma 1.3 (2), (8) that

$$a \wedge x \leq a \wedge (c \oplus b) \leq (a \wedge c) \oplus (a \wedge b) = 0$$

Hence $a \in Ann(I \vee J)$. Thus $Ann(I \vee J) \supseteq Ann(I) \cap Ann(J)$. □

Lemma 2.17. *Let A be involutive. We have*

$$Ann(I) \vee Ann(J) \subseteq Ann(I \cap J).$$

Proof. Since A is involutive, by Lemma 2.16(iii) and (iv), we have

$$\begin{aligned} I \cap J &= Ann(Ann(I \cap J), \\ &= Ann(Ann(I) \cap Ann(J)), \\ &= Ann(Ann(I)) \cap Ann(Ann(J)), \\ &\subseteq Ann(Ann(I) \vee Ann(J)). \end{aligned}$$

Hence $Ann(Ann(I \cap J)) \subseteq Ann(Ann(I) \vee Ann(J))$, so by Lemma 2.16(ii), we get $Ann(Ann(Ann(I) \vee Ann(J))) \subseteq Ann(Ann(Ann(I \cap J)))$. Thus we obtain $Ann(I) \vee Ann(J) \subseteq Ann(I \cap J)$. □

Theorem 2.18. *Let A be an involutive MV-algebra. Then, every non-zero proper ideal of A is finitely meet-irreducible ideal if and only if it is a soaker ideal of A.*

Proof. Suppose that P is a soaker ideal of A and $I \cap J \subseteq P$, where $I, J \in Id(A)$. It follows from Lemma 2.16(iii) and (iv) that

$$\begin{array}{rcl} Ann(P) & \subseteq & Ann(I \cap J), \\ & = & Ann(I) \cap Ann(J), \\ & \subseteq & Ann(I) \vee Ann(J). \end{array}$$

Since $Ann(P)$ is a soaker ideal, $Ann(P) \subseteq Ann(I)$ or $Ann(P) \subseteq Ann(J)$. Then by Lemma 2.16(ii), we get $Ann(Ann(P)) \supseteq Ann(Ann(I))$ or $Ann(Ann(P)) \supseteq Ann(Ann(J))$. Then we obtain $P \supseteq I$ or $P \supseteq J$.

Conversely, suppose that P is a meet-irreducible ideal of A. Let $P \subseteq I_1 \vee I_2 \vee \cdots \vee I_k$, where $I_i \in Id(A)$, for $i = 1, \cdots k$. It follows from Lemma 2.16(ii) and (iv) that

$$Ann(P) \supseteq Ann(I_1 \vee \cdots \vee I_k) \supseteq Ann(I_1) \cap \cdots \cap Ann(I_k).$$

Since $Ann(P)$ is an ideal, so by hypothesis, $Ann(P) \supseteq Ann(I_1)$ or $Ann(P) \supseteq Ann(I_2)$,..., $Ann(P) \supseteq Ann(I_k)$. Now, by Lemma 2.16(ii), we get $Ann(Ann(P)) \subseteq Ann(Ann(I_1))$ or ....or $Ann(Ann(P)) \subseteq Ann(Ann(I_k))$. We obtain $P \subseteq I_1$ or ... or $P \subseteq I_k$. □

Theorem 2.19. *If an ideal P is soaker, then $Ann(P)$ is finitely meet-irreducible.*

Proof. Suppose P is soaker ideal and $I \cap J \subseteq Ann(P)$. It follows from Lemma 2.16(ii) that $Ann(Ann(P)) \subseteq Ann(I \cap J)$. By Lemma 2.16(i), (iii) and (iv), we have

$$\begin{array}{rcl} P & \subseteq & Ann(Ann(P)), \\ & \subseteq & Ann(I \cap J), \\ & = & Ann(I) \cap Ann(J), \\ & \subseteq & Ann(I) \vee Ann(J). \end{array}$$

Since P is a soaker ideal, $P \subseteq Ann(I)$ or $P \subseteq Ann(J)$. By Lemma 2.16(ii), we have $Ann(Ann(I)) \subseteq Ann(P)$ or $Ann(Ann(J)) \subseteq Ann(P)$. It follows from Lemma 2.16(i) that $I \subseteq Ann(P)$ or $J \subseteq Ann(P)$. □

Theorem 2.20. *If A is involutive and $Ann(P)$ is finitely meet-irreducible, then the ideal P is soaker.*

Proof. Suppose that P is finitely meet-irreducible and $P \subseteq I_1 \vee I_2 \vee \cdots \vee I_k$, where $I_i \in Id(A)$, for $i = 1, \cdots k$. Hence, by Lemma 2.16(ii) and (iv), we get

$$Ann(P) \supseteq Ann(I_1 \vee \cdots \vee I_k) \supseteq Ann(I_1) \cap \cdots \cap Ann(I_k).$$

Since $Ann(P)$ is finitely meet-irreducible, $Ann(P) \supseteq Ann(I_1)$ or $Ann(P) \supseteq Ann(I_2)$,..., $Ann(P) \supseteq Ann(I_k)$. Now, by Lemma 2.16(ii), we get $Ann(Ann(P)) \subseteq Ann(Ann(I_1))$ or ....or $Ann(Ann(P)) \subseteq Ann(Ann(I_k))$. We obtain $P \subseteq I_1$ or ... or $P \subseteq I_k$. $\square$

By Theorem 2.18 and Lemma 1.6, we obtain the following corollary:

Corollary 2.21. *Let A be an involutive MV-algebra. Then P is a soaker ideal if and only if P is a prime ideal.*

3 The reflectional topology in MV-algebras

Let A be an MV-algebra. We denote the set of all soaker ideals of A by $Refl(A)$ and for $I \in Id(A)$, we define $U(I) = \{S \in Refl(A) \mid S \subseteq I\}$.

Proposition 3.1. *Let A be an MV-algebra. Then for ideals I, K and $\{I_i\}_{i \in J} \subseteq Id(A)$, we have:*
(i) $U(A) = Refl(A)$ *and* $U(\{0\}) = \emptyset$.
(ii) $I \subseteq K$ *implies* $U(I) \subseteq U(K)$.
(iii) $\bigcap_{i \in J} U(I_i) = U(\bigcap_{i \in J} I_i)$.
(iv) $\bigcup_{i \in J} U(I_i) = U(\bigvee_{i \in J} I_i)$.

Proof. (i) Obviously, $U(A) = Refl(A)$ and $U(\{0\}) = \emptyset$.

(ii) By definition, it is clear.

(iii) Since $\bigcap_{i \in J} I_i \subseteq I_i$, for all $i \in J$, by part (ii), $U(\bigcap_{i \in J} I_i) \subseteq U(I_i)$, for all $i \in J$. Hence $U(\bigcap_{i \in J} I_i) \subseteq \bigcap_{i \in J} U(I_i)$. Conversely, let $S \in \bigcap_{i \in J} U(I_i)$. Then $S \in U(I_i)$, for all $i \in J$. We get $S \subseteq F_i$, for all $i \in I$, and so $S \subseteq \bigcap_{i \in I} F_i$. Thus $S \in U(\bigcap_{i \in J} I_i)$.

(iv) Suppose that $S \in \bigcup_{i \in J} U(I_i)$. Then we have $S \in U(I_i)$, for some $i \in J$. Hence $S \subseteq I_i$, for some $i \in J$. We get $S \subseteq I_i \subseteq \bigvee_{i \in J} I_i$. It follows that $S \in U(\bigvee_{i\in J} I_i)$. Thus $\bigcup_{i\in J} U(I_i) \subseteq U(\bigvee_{i\in J} I_i)$. Conversely, suppose that $S \in U(\bigvee_{i\in J} I_i)$. We get $S \subseteq \bigvee_{i\in J} I_i$. Since S is a soaker ideal, we get $S \subseteq I_j$, for some $j \in J$. Hence $S \in U(I_j)$, for some $j \in J$. We obtain $S \in \bigcup_{j\in J} U(I_j)$. Therefore $\bigcup_{i\in J} U(I_i) = U(\bigvee_{i\in J} I_i)$. □

By parts (i), (iii) and (iv) of Proposition 3.1, $\tau_X = \{U(I) | I \in Id(A)\}$ is a topology on the set $X = Refl(A)$, called *the reflectional topology.*

Recalling that an alexandrov topological space is one in which the intersection of any collection of open sets is open, or equivalently the union of any collection of closed sets is closed, we have:

Theorem 3.2. *(X, τ_X) is an Alexandrov topology.*

Proof. The proof follows from part (iii) of Proposition 3.1. □

Example 3.3. Consider Example 2.2. We have $U(I_1) = \emptyset$, $U(I_2) = \{I_2\}$, $U(I_3) = \{I_3\}$, and $U(A) = \{I_2, I_3\}$. Hence we obtain $\tau_X = \{\emptyset, \{I_2\}, \{I_3\}, X\}$.

Example 3.4. Consider Example 2.3.We can easily check that $U(I_0) = \emptyset$, $U(I_1) = \{I_1\}$, $U(I_2) = \{I_2\}$, and $U(A) = \{I_1, I_2\}$. Hence we obtain $\tau_X = \{\emptyset, \{I_1\}, \{I_2\}, X\}$.

Example 3.5. Consider Example 2.4. We can easily check that $U(I_0) = \emptyset$, $U(I_1) = \{I_1\}$, $U(I_2) = \{I_2\}$, $U(I_3) = \{I_3\}$, $U(I_4) = \{I_1, I_2\}$, $U(I_5) = \{I_1, I_3\}$, $U(I_6) = \{I_2, I_3\}$, and $U(A) = X$. We obtain

$$\tau_X = \{\emptyset, \{I_1\}, \{I_2\}, \{I_3\}, \{I_1, I_2\}, \{I_1, I_3\}, \{I_2, I_3\}, X\}.$$

Example 3.6. Consider Example 2.5. It is clear that $X = Refl(A) = \{P_i/i \in \mathbb{N}\}$ and $U(\{(0, \mathbf{0}) = \emptyset$, $U(P_1) = \{P_i\}_{i\geq 1}$, $U(P_2) = \{P_i\}_{i\geq 2}$ and $U(P_n) = \{P_i\}_{i\geq n}$. Therefore we obtain $\tau_X = \{\emptyset, X, \{P_i\}_{i\geq 1}, \{P_i\}_{i\geq 2}, \cdots\}$ is topology.

For $a \in A$, let us denote $U((a])$ by $U(a)$. So $U(a) = \{S \in X \mid S \subseteq (a]\}$.

Proposition 3.7. *Let $a, b \in A$. Then*

(i) *if $a = 0$, then $U(a) = \emptyset$.*
(ii) *if $(a] = (b]$, then $U(a) = U(b)$.*
(iii) *if $a \leq b$, then $U(a) \subseteq U(b)$.*
(iv) $U(a) \cap U(b) = U(a \wedge b)$.
(v) $U(a) \cup U(b) = U(a \vee b) = U(a \oplus b)$.

Proof. (i) and (ii) are clear.

(iii) Suppose that $a \leq b$. It follows from Remark 1.9(4) that $(a] \subseteq (b]$. Let $S \in U(a)$. We get $S \subseteq (a] \subseteq (b]$. We conclude that $S \in U(b)$. Thus $U(a) \subseteq U(b)$.

(iv) By Remark 1.9(3), we have

$$\begin{aligned} S \in U(a) \cap U(b) &\Leftrightarrow S \subseteq (a] \text{ and } S \subseteq (b] \\ &\Leftrightarrow S \subseteq (a] \wedge (b] \\ &\Leftrightarrow S \subseteq (a \wedge b] \\ &\Leftrightarrow S \in U(a \wedge b). \end{aligned}$$

(v) It follows from Lemma 2.7 that

$$\begin{aligned} S \in U(a) \cup U(b) &\Leftrightarrow S \subseteq (a] \text{ or } S \subseteq (b] \\ &\Leftrightarrow S \subseteq (a] \vee (b] (\text{ since } (a], (b] \subseteq (a] \vee (b]) \\ &\Leftrightarrow S \subseteq (a \vee b] \\ &\Leftrightarrow S \subseteq (a \oplus b] \\ &\Leftrightarrow S \in U(a \vee b) \text{ and } S \in U(a \oplus b). \end{aligned}$$

Conversely, let $S \in U(a \oplus b)$. Then we get $S \subseteq (a \oplus b]$. It follows from Lemma 2.7 that $S \subseteq (a] \vee (b]$. Since S is a soaker ideal, $S \subseteq (a]$ or $S \subseteq (b]$. Thus $S \in U(a) \cup U(b)$. □

Lemma 3.8. *Any open subset of X is a union of subsets from the family $\{U(a)|a \in A\}$.*

Proof. An open subset of X is of the form $U(I)$, for an ideal I of A. It follows from Proposition 3.1(iv) that $U(I) = U(\bigvee_{a\in I}(a]) = \bigcup_{a\in I} U((a]) = \bigcup_{a\in I} U(a)$, as desired. □

Theorem 3.9. *Let A be an MV-algebra. The family $\{U(a)|a \in A\}$ is a basis for the topology of X.*

Proof. By Proposition 3.1(iii), $\{U(a)|a \in A\}$ is closed under intersection and by Lemma 3.8, $X = U(A) = \cup_{a\in A} U(a)$. The result then follows. □

The set $U(a)$ will be called a basic open set of X.

4 Compactness and connectedness

Considering X as a partially ordered set under inclusion, we have:

Lemma 4.1. *Let I, J be ideals of an MV-algebra A. J is an upper bound of $U(I)$ if and only if $U(I) \subseteq U(J)$.*

Proof. Suppose that J is an upper bound of $U(I)$. So for all $S \in U(I)$, $S \subseteq J$. Hence for all $S \in U(I)$, $S \in U(J)$. Thus we get $U(I) \subseteq U(J)$. Conversely, suppose $U(I) \subseteq U(J)$. So for every $S \in U(I)$, $S \in U(J)$. Hence for every $S \in U(I)$, $S \subseteq J$. Hence J is an upper bound of $U(I)$. □

Corollary 4.2. *I is an upper bound of $U(I)$.*

Denoting "S is a soaker ideal and $S \subseteq I$" by $S \subseteq_s I$, we have:

Lemma 4.3. *$\bigvee_{S \subseteq_s I} S$ is a least upper bound of $U(I)$.*

Proof. Let $S \in U(I)$. Then $S \subseteq_s I$, and so $S \subseteq \bigvee_{S \subseteq_s I} S$. So $\bigvee_{S \subseteq_s I} S$ is an upper bound of $U(I)$. Suppose J is an upper bound of $U(I)$. Then for all $S \in U(I)$, $S \subseteq J$. Hence we get $\bigvee_{S \subseteq_s I} S \subseteq J$. So $\bigvee_{S \subseteq_s I} S$ is a least upper bound of $U(I)$. □

Lemma 4.4. *$U(I) = U(\bigvee_{S \subseteq_s I} S)$.*

Proof. Since $\bigvee_{S \subseteq_s I} S \subseteq I$, $U(\bigvee_{S \subseteq_s I} S) \subseteq U(I)$. If $S \in U(I)$, then $S \subseteq_s I$. So $S \subseteq_s \vee_{S \subseteq_s I} S$. Thus $S \in U(\bigvee_{S \subseteq_s I} S)$. Therefore $U(I) \subseteq U(\bigvee_{S \subseteq_s I} S)$. □

Definition 4.5. An ideal I is said to be a join-soaker ideal (or JS-ideal), if I is the least upper bound of $U(I)$.

Note that any soaker ideal is a JS-ideal.

Lemma 4.6. *For an ideal I, the following conditions are equivalent:*

(1) *For all $J \in Id(A)$, $U(I) \subseteq U(J)$ implies $I \subseteq J$.*

(2) *I is a join-soaker ideal.*

(3) *$F = \bigvee_{S \subseteq_s I} S$.*

Proof. (1) ⇒ (2) By Corollary 4.2, I is an upper bound of $U(I)$. Let J be an upper bound of $U(I)$. By Lemma 4.1, we get $U(I) \subseteq U(J)$. By hypothesis, $I \subseteq J$. Hence I is the least upper bound of $U(I)$.

(2) ⇒ (3) By Lemma 4.3, $\bigvee_{S \subseteq_s I} S$ is a least upper bound of $U(I)$. By hypothesis, I is the least upper bound of $U(I)$. Therefore $I = \bigvee_{S \subseteq_s I} S$.

(3) ⇒ (1) Suppose $U(I) \subseteq U(J)$. By Lemma 4.1, J is an upper bound of $U(I)$. By hypothesis, $I = \bigvee_{S \subseteq_s I} S$ and by Lemma 4.3, $\bigvee_{S \subseteq_s I} S$ is the least upper bound of $U(I)$. So I is the least upper bound of $U(I)$. Thus $I \subseteq J$. □

Remark 4.7. We can easily show that if $(a] \subseteq \bigvee_\alpha (b_\alpha]$, then $(a] \subseteq \bigvee_{i=1}^k (b_i]$. Also we have, if $(a] = \bigvee_\alpha (b_\alpha]$, then $(a] = \bigvee_{i=1}^k (b_i]$.

Lemma 4.8. *$(a]$ is a JS-ideal if and only if there exists $b_1, b_2, \cdots, b_k$ satisfiying:*

(1) *for all $i \in I$, $b_i \in C$,*

(2) *there exists m such that for all $i \in I$, $b_i \leq ma$, and*

(3) *$b_1 \oplus b_2 \oplus \cdots \oplus b_k \geq a$.*

In such a case $(a] = \bigvee_{i=1}^n (b_i]$, where $(b_i]$ is soaker and $(b_i] \subseteq (a]$.

Proof. Suppose that $(a]$ is a JS-ideal. Since soaker ideals are principal, $(a] = \vee_{(b_\alpha] \subseteq (a]} (b_\alpha]$, where $(b_\alpha]$'s are soaker ideals of A. Hence $a \leq n_1 b_{\alpha_1} \oplus n_2 b_{\alpha_2} \oplus \cdots \oplus n_k b_{\alpha_k}$. If there is no soaker ideal contained in $(a]$, then $(a] = \bigvee (b_\alpha] = \emptyset$, which is a contradiction. So $k \geq 1$. Let $b_1 = n_1 b_{\alpha_1}$, $b_2 = n_2 b_{\alpha_2}$, $\cdots$. Then, since $b_{\alpha_i} \in C$, by Lemma 2.10, we have $b_i \in C$. On the other hand, since $(b_{\alpha_i}] \subseteq (a]$, for all $i \in J$, $b_{\alpha_i} \leq am_i$. Let $m = max\{m_i : i = 1, \cdots, k\}$. Then for all $i \in J$, $b_{\alpha_i} \leq am$. Finally, $b_1 \oplus \cdots \oplus b_k = n_1 b_{\alpha_1} \oplus \cdots n_k b_{\alpha_k} \geq a$.

Conversely, let there exist $b_1, b_2, \cdots, b_k$ satisfing conditions (1), (2) and (3). By (1), $(b_i]$ is soaker. By (2), $b_i \leq ma$ and so $(b_i] \subseteq (a]$. By (3), $a \leq b_1 \oplus \cdots \oplus b_k$. Therefore $(a] \subseteq \bigvee_{i=1}^k (b_i]$. We conclude that $(a] = \bigvee_{i=1}^k (b_i]$, with $(b_i]$'s soaker. Thus $(a] = \bigvee_{(b_\alpha] \subseteq (a]} (b_\alpha]$, where $(b_\alpha]$'s are soaker ideals. Hence $(a]$ is a JS-ideal. □

Proposition 4.9. *Let $a \in A$. If $(a]$ is a JS-ideal, then $U(a)$ is a compact subset of X.*

Proof. It is sufficient to show that any cover of $U(a)$ by basic open sets contains a finite subcover. Let $U(a) \subseteq \bigcup_{i \in I} U(a_i)$. By Proposition 3.1, we have $U(a) \subseteq \bigcup_{i\in I} U((a_i]) = U(\bigvee_{i\in I}((a_i]))$. By Lemma 4.6, we have $\langle a\rangle \subseteq \vee_{i\in I}\langle a_i\rangle$ and so $a \in \vee_{i\in I}(a_i]$. It follows from Remark 1.9 that $a \leq n_1 a_{i_1} \oplus \cdots \oplus n_k a_{i_k}$. By parts (iii) and (v) of Proposition 3.7, we have $U(a) \subseteq U(n_1 a_{i_1} \oplus \cdots \oplus n_k a_{i_k}) = U(a_{i_1}) \cup \cdots \cup U(a_{i_k})$, as desired. □

Corollary 4.10. *If A is a JS-ideal, then X is compact.*

Proof. Since $(1] = A$, by hypothesis $(1]$ is a JS-ideal. Now by Theorem 4.9, $X = U(A) = U(1)$ is compact. □

Denoting "S is a soaker ideal and $S \not\subseteq I$" by $S \not\subseteq_s I$, we have:

Theorem 4.11. *For any ideal I, $X - U(I) \subseteq U(\bigvee_{S\not\subseteq_s I} S)$.*

Proof. Let $S \in X - U(I)$. Then $S \not\subseteq I$, hence $S \subseteq \bigcup_{S\not\subseteq_s I} S \subseteq \bigvee_{S\not\subseteq_s I} S$. So $S \in U(\bigvee_{S\not\subseteq_s I} S)$. □

Theorem 4.12. *Let I, K be ideals of A. $X - U(I) \subseteq U(K)$ if and only if $\bigvee_{S\not\subseteq_s I} S \subseteq K$.*

Proof. Suppose $X - U(I) \subseteq U(K)$. Then for all $S \in X - U(I)$, $S \in U(K)$, that is, for all $S \in X$, $S \not\subseteq I$, $S \subseteq K$. So $\bigcup_{S\not\subseteq_s I} S \subseteq K$, hence $\bigvee_{S\not\subseteq_s I} S \subseteq K$. Conversely, suppose $\vee_{S\not\subseteq I} S \subseteq K$. So $U(\bigvee_{S\not\subseteq I} S) \subseteq U(K)$. By Theorem 4.11, we get $X - U(I) \subseteq U(K)$. □

Theorem 4.13. *$U(I)$ is closed if and only if $X - U(I) = U(\bigvee_{S\not\subseteq_s I} S)$.*

Proof. Suppose $U(I)$ is closed. So there exists an ideal K of A such that $X - U(I) = U(K)$. It follows from Theorem 4.12 that $\bigvee_{S\not\subseteq_s I} S \subseteq K$ and $U(K) \subseteq X - U(I)$. So, $U(\bigvee_{S\not\subseteq_s I} S) \subseteq U(K)$ and $U(K) \subseteq X - U(I)$. This yields $U(\bigvee_{S\not\subseteq_s I} S) \subseteq X - U(I)$. Now, by Theorem 4.11, we have $X - U(I) = U(\bigvee_{S\not\subseteq_s I} S)$.

Conversely, suppose that $X - U(I) = U(\vee_{S\not\subseteq_s I} S)$. Since $U(\bigvee_{S\not\subseteq_s I} S)$ is open, $X - U(I)$ is open. Thus $U(I)$ is closed. □

Since $U(I)$ is open in X, the above theorem characterizes the clopen (that is, closed and open) subsets of X. In the following examples, we show which subsets are clopen.

Theorem 4.14. *(X, τ_X) is connected if and only if for all $I \in Id(A)$, if $X - U(I) = U(\bigvee_{S \not\subseteq_s I} S)$, then either no soaker ideal is contained in I or all the soaker ideals are contained in I.*

Proof. (X, τ_X) is connected if and only if the only clopen subsets of X are $\emptyset$ or X if and only if for all ideals I, if $X - U(I) = U(\bigvee_{S \not\subseteq_s I} S)$, then $U(I) = \emptyset$ or $U(I) = X$. Since $U(I) = \emptyset$ if and only if I contains no soaker ideal and $U(I) = X$ if and only if I contains all the soaker ideals, the result follows. □

Example 4.15. Consider Example 2.5. Using Example 3.6, we have $U(P_k)^c = \{P_k, P_{k+1}, \cdots\}^c = \{P_1, P_2, \cdots, P_{k-1}\} \neq U(J)$, for all $J \in Id(A)$, we get $U(P_k)$ is not closed, for all $k \in N$. Hence X is connected.

Example 4.16. Consider Example 2.2. Since $U(I_2)$ and $U(I_3)$ are closed and open, hence X is disconnected.

Proposition 4.17. *Let A be a Boolean algebra. Then for each $a \in A$, $U(a)$ is a clopen subset of X.*

Proof. By Proposition 3.7, we have $U(a) \cup U(a') = U(a \oplus a') = U(1) = X$ and $U(a) \cap U(a') = U(a \wedge a') = U(0) = \emptyset$. So $U(a) = X - U(a')$ and therefore it is closed. We know $U(a)$ is also open, thus it is a clopen subset of X. □

Corollary 4.18. *Let A be a Boolean algebra. Then for each $I \in Id(A)$, $U(I)$ is a clopen subset of X.*

Proof. Since $I = \bigvee_{a \in I}(a]$, the proof follows from Propositions 3.1, 3.2 and 4.17. □

Theorem 4.19. *Let A be a Boolean algebra. (X, τ_X) is connected if and only if X contains at most one element.*

Proof. If X is connected, then for each $I \in Id(A)$, either I contains no soaker ideal or it contains all the soaker ideals. Now if S is a soaker ideal, then S must contain all the soaker ideals. It follows that if S and S' are two soaker ideals, then $S = S'$. Hence there is at most one soaker ideal of A and so X contains at most one element. The converse is obvious. □

Theorem 4.20. *The topological space* (X, τ_X) *is a* T_0*-space.*

Proof. Let $S \neq S'$ be soaker ideals of A. Then $S \notin U(S')$ or $S' \notin U(S)$. Also, we have $S \in U(S)$ and $S' \in U(S')$. Therefore $U(S)$ or $U(S')$ contains only one of the ideals S or S'. Hence X is a T_0-space. □

Proposition 4.21. *The topological space* (X, τ_X) *is a* T_1*-space if and only if for all soaker ideals* S, $U(S) = \{S\}$.

Proof. Suppose that X is T_1 and let $S \in X$. For $S' \in U(S)$ if $S' \neq S$, then there exists an ideal I of A such that $S \in U(I)$ and $S' \notin U(I)$. Hence, we get $U(S) \subseteq U(I)$ and $S' \notin U(I)$. Thus $S' \notin U(S)$, which is a contradiction. Therefore, $S' = S$. Hence $U(S) = \{S\}$. Conversely, suppose for all soaker ideals S, $U(S) = \{S\}$. Given soaker ideals $S \neq S'$, $U(S) = \{S\}$ is a neighborhood of S not containing S' and $U(S') = \{S'\}$ is a neighborhood of S' not containing S. Thus X is T_1. □

Theorem 4.22. *The topological space* (X, τ_X) *is a* T_2*-space if and only if it is* T_1.

Proof. We know every T_2-space is T_1. The converse follows easily from Proposition 4.21. □

Example 4.23. Consider Example 2.4. Using Example 3.5, since for any soaker ideal I_i, we get $U(I_i) = I_i$, for any $i = 1, 2, 3$. So by Proposition 4.21, X is a T_1-space and thus T_2.

Example 4.24. Consider Example 2.5. Using Example 3.6, since $U(P_k) = \{P_i\}_{i \geq k}$, for $k \in N$, by Proposition 4.21, X is not a T_1-space and thus it is not T_2.

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