



# Categories and General Algebraic Structures with Applications

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# Categories and General Algebraic Structures with Applications

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## Aims and Scope:

“**Categories and General Algebraic Structures with Applications**” is an international journal published by Shahid Beheshti University, Tehran, Iran, **free of page charges**. It publishes original high quality research papers and invited research and survey articles mainly in two subjects: **Categories** (algebraic, topological, and applications in mathematics and computer sciences) and **General Algebraic Structures** (not necessarily classical algebraic structures, but universal algebras such as algebras in categories, semigroups, their actions, automata, ordered algebraic structures, lattices (of any kind), quasigroups, hyper universal algebras); and their applications.

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