



## An interview with George A. Grätzer

Gábor Czédli

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This interview was conducted in the second half of May, 2018; both George Grätzer and the author were at home, in Toronto and Szeged, respectively. They communicated via a lot of e-mails and a few phone calls.

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G.Cz. *Was there any tradition of mathematics in your family?*

G.G. Not directly. However, my father, József, was known as the “King of Puzzles” (Rejtvény Király), who made a living creating puzzles. He introduced crossword puzzles to Hungary. His puzzle column was syndicated in 250 newspapers.

G.Cz. *When and how did your mathematical talent manifest itself?*

G.G. I was a grade 11 student, it was almost the end of the academic year. It was a Friday, and I was visiting a distant relative, a retired

math teacher. His apartment was littered with old copies of KML (Középiskolai Matematikai Lapok—Highschool Mathematical Journal). To my surprise, there was a current issue. Throughout the academic year, KML published a set of problems for students (separate sets for grades 9–12) with deadlines for solutions. The deadline in this issue was the following Monday. I worked on the problems that weekend, and mailed in some solutions on Monday. To my surprise, I ended up as 24th in the annual competition of about 400 students. For grade 12, I subscribed to KML and submitted the problem solutions every month. Got the third prize and an incentive to study mathematics.

G.Cz. *What was your motivation to start your university studies as a student of mathematics?*

G.G. Through the KML competition, I got to know a number of grade 12 students active in mathematics. I proposed to form the Youth Wing of the Hungarian Mathematical Society (few years later it was recognized officially by the Society), and we arranged a series of lectures by prominent mathematicians including the geometer László Fejes Tóth, the author of the famous book, *Lagerungen in der Ebene, auf der Kugel und im Raum* (Springer-Verlag, Berlin-New York, 1972). Some of his results used integral geometry, so he could give no proofs to grade 12 students. E.T. Schmidt (Tomi) sat next to me, and we started whispering that we can prove a slightly weaker version of one of his results, using elementary geometry. After the lecture we presented this to Fejes Tóth, who encouraged us to write the proof up in a paper. This was my first mathematical paper, and the first of more than 60 joint papers with Tomi.

G.Cz. *Your first publications, at age 18–20, appeared on chess problems (endgames and helpmates), not on mathematics. Are you still interested in chess and chess problems? Do you play chess regularly?*

G.G. My story with chess and chess compositions is told on my website:

`server.math.umanitoba.ca/~gratzer/`

I still play chess as a relaxation on the Web site chess.com.

G.Cz. *When did you meet E. Tamás Schmidt first and how did your friendship and exceptionally fruitful collaboration start?*

G.G. As I related before, we met as grade 12 students. After graduating from high school, we decided to read math books. We started with János Bolyai's *Appendix*, a treatise on non-Euclidean geometry. Not a very good choice.

As second year math students, we got permission to attend László Fuchs' special course on ordered sets and lattices. That is how we found Garrett Birkhoff's *Lattice Theory* (American Mathematical Society, New York, N. Y., 1940) and started doing research by solving open problems proposed in the book.

G.Cz. *After you moved to the United States and later to Canada, how did you work with Tamás (Schmidt) before the Internet?*

G.G. I did not. On the advice of his wife's family, he stopped collaborating with me in the fall of 1960. In those days, it took about two years (from the submission) to publish a paper. The papers that were published after 1960 were all written in 1960 or before. The only exception was our paper characterizing congruence lattices of algebras that took me a long time to write up. I completed it after I returned from Canada where I was a postdoctoral fellow in 1961.—Thirty years passed before we collaborated again, in person, in Winnipeg. For more detail see my article

<https://doi.org/10.1007/s00012-018-0485-0>

and in the listing of my papers on Research Gate:

<https://www.researchgate.net/profile/>

[George\\_Graetzer/contributions](https://www.researchgate.net/profile/George_Graetzer/contributions)

G.Cz. *If you were at age 20 or so, would you choose lattice theory again? Would you recommend it to others at the beginning of their scientific career now?*

- G.G. In 1965, Paul Erdős was discussing math with a group of very talented highschool students—including László Lovász, who became the President of IMU (2007–2010) and the President of the Hungarian Academy of Sciences (2014–). Out of the blue, one of them asked, which branch of math should we choose to make the most money. In typical Erdős fashion, he answered in a completely serious fashion: statistics is the best paid. (Computer science did not yet exist. A short while ago, an AI expert got a new job paying two million dollars.)—I have always felt that you should not become a mathematician unless there is a burning desire in your stomach telling you that there is nothing else you can do. If I was 20 again, probably I would try to invent Computer Science and make a lot of money.
- G.Cz. *How do you see the future of lattice theory? And that of “pure lattice theory”, that is, the theory of lattices without extra operations?*
- G.G. I do not see the future, just the beauty of the results I am trying to find.
- G.Cz. *In 1968, your first mathematical book was titled “Universal Algebra”. In 1971, you founded a highly important journal under the name “Algebra Universalis”; it is still flourishing. Where was lattice theory? Why lattices are not included in the name of your journal?*
- G.G. A linguist kindly helped me to say Universal Algebra in Latin. There was no way to translate Lattice Theory.
- G.Cz. *As a continuation of my previous question, a few years later, you published more papers in lattice theory than in universal algebra. For the last decade, I would say that 90–95 percent of your research interest is aimed at lattices rather than at universal algebra; do you agree? Why did you give preference to lattices?*
- G.G. I started out in the early sixties with two goals for writing monographs: Universal Algebra and Lattice Theory. I started with Universal Algebra—I do not remember why—and when this first monograph was finished, around 1967, I started doing the research for the Lattice Theory book. This was a big project. With my seminar (meeting

three to six hours a week), we went through a very large number of papers, discarding many. It took me more than ten years to finish the book, and the process spawned hundreds of research projects. (See my paper, General Lattice Theory: 1979 Problem Update, *Algebra Universalis* **11** (1980), 396–402.) I think that I got stuck in these projects for another decade or two.

G.Cz. *For most mathematicians, you are known as the author of many clear and readable books on L<sup>A</sup>T<sub>E</sub>X. What was your motivation to become an expert of L<sup>A</sup>T<sub>E</sub>X and to write these books?*

G.G. I was in Kyoto at the IMU meeting when AMS-L<sup>A</sup>T<sub>E</sub>X was released. What a pleasure it was to see professional quality mathematical typesetting. The booklet telling us how to use AMS-L<sup>A</sup>T<sub>E</sub>X was clear but almost useless. It said: 1. Read a book on L<sup>A</sup>T<sub>E</sub>X. But remember this is not L<sup>A</sup>T<sub>E</sub>X. 2. Read a book on AMS-T<sub>E</sub>X. But remember this is not AMS-T<sub>E</sub>X. 3. The following pages describe how AMS-L<sup>A</sup>T<sub>E</sub>X is different from L<sup>A</sup>T<sub>E</sub>X and AMS-T<sub>E</sub>X. And I said, there must be a better way. Hence my first book on L<sup>A</sup>T<sub>E</sub>X—really, AMS-L<sup>A</sup>T<sub>E</sub>X.

G.Cz. *As the author of several books on L<sup>A</sup>T<sub>E</sub>X, are you treated better by technical editors of mathematical journals than us, the rest of the authors?*

G.G. No. Just the opposite. If I try to help, they resent it.

G.Cz. *Could you say a few words to mathematicians outside lattice theory about what the beauty of this theory is?*

G.G. No. But I could recommend some papers to read.

G.Cz. *Can you recommend three of your papers?*

G.G. Here are three:

1. *Two problems that shaped a century of lattice theory.* Notices Amer. Math. Soc. **54** (2007), 696–707.

Just read the section on Uniquely Complemented Lattices. Such a difficult problem, and a super simple visual approach.

2. *The order of principal congruences of a lattice.* Algebra Universalis **70** (2013), 95–105.

A two-line result with a really visual proof.

3. *Congruences in slim, planar, semimodular lattices: The Swing Lemma,* Acta Sci. Math. (Szeged) **81** (2015), 381–397.

A very effective, easy to visualize description of congruences in this class of lattices.

G.Cz. *What would you add to your famous results I singled out in the accompanying paper?*

G.G. Major topics and results:

1. Congruence lattices of finite lattices. I wrote more than 50 papers in this field, many of them with Tomi. An overview of this field is given in my book *The Congruences of Finite Lattices*, second edition (Birkhäuser, 2016).

2. *The Amalgamation Property in equational classes of modular lattices.* Pacific J. Math. **45** (1973), 507–524 with B. Jónsson and H. Lakser. In this paper we prove that there are only three varieties of modular lattices with the Amalgamation Property, the three obvious ones.

3. *The Strong Independence Theorem for automorphism groups and congruence lattices of arbitrary lattices.* Advances in Applied Mathematics **24** (2000), 181–221 with F. Wehrung. The title says it all.

4. *Complete congruences of complete lattices.* This field started with a paper of mine in 1989 and culminated in the result you mention in your piece. There are about a dozen articles on this topic.

5. *The ordered set of principal congruences.* You describe this topic in your accompanying paper.

These major topics cover fewer than 100 of my papers. I apologize to all those I left out.

G.Cz. *The work you have completed so far (including the L<sup>A</sup>T<sub>E</sub>X books, being a founding editor-in-chief of a journal for 45 years, four monographs on mathematics and their extended and/or revised versions, 14 Ph.D.*

*theses under your supervision, and about 260 mathematical papers) is really impressive. Another aspect of my experience with you is that whatever you do, you do it very efficiently. Is there any secret, like talent, hard work, good methods, being well-organized, support from family, sport, etc. of your productivity that you can share with us?*

G.G. I think it's simple: you work to satisfy yourself, as best as you can. Keep your theorems short, only the proofs should be long.

As for efficiency, there may be one secret. I try out *everything* that may help. Text expanders, apps for the Mac . . . Some do help.

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