Categories and General Algebraic Structures with Applications

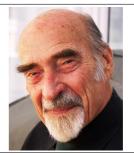


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## An interview with Bernhard Banaschewski

Christopher Gilmour

This interview, a co-operative effort of Bernhard Banaschewski and Christopher Gilmour, took place over a few days in December, 2016. It was finalised over coffee and a shared slice of excellent cheesecake at *The Botanical Tea Garden*, a small, home situated, tea garden in Little Mowbray, Cape Town.



## Interview

- **C.G.** Your doctoral studies under Ernst Witt, who was one of the leading algebraists at the time, were in topology; how did this come about?
- **B.B.** After the end of the war in 1945, Ernst Witt was considering writing a book about topological groups and, in preparation for that, he decided to study Bourbaki's *Topologie Generale*, one of the most recent texts on the subject and certainly the most innovative stylistically. So, in 1951, when I discussed with him what I might do for my diploma

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thesis (for which he had agreed to be my supervisor), he suggested I, too, should study Bourbaki and see what I could find to work on. As a result that thesis (1952) dealt with spaces of filters (the latter being Bourbaki's new tool in topology), such as filter spaces on sets, and my doctoral dissertation (1953) then made systematic use of filter spaces to describe the extensions of any given topological space. Incidentally, this work was very much influenced by P. Samuel's "Ultrafilters and compactification of uniform spaces", which I later on learned was criticized in some French circles as "ultra-Bourbaki".

- C.G. Your early research years were at a time when the influence of Bourbaki and the conceptual approach to mathematics was growing. Your love for conceptual thinking is evident in all you do. What drew you towards this end of the spectrum of mathematical topics?
- **B.B.** Witt was very much concerned that mathematics should be presented in the conceptually most efficient way. This was what attracted him to Bourbaki, and I think I simply took that over from him.
- C.G. Your very first paper, Über den Satz von Zorn, was published in the same year as your doctorate You have had an abiding and critical interest in choice principles, and their rôle in a wide variety of situations, ever since. Was this what drew you to topos theory and pointfree topology?
- **B.B.** Regarding that first paper: it was a follow-up to a paper by Witt in which I somehow succeeded to streamline his arguments. My interest in choice principles and other foundational questions was basically driven by Witt's interest in Brouwerian "intuitionism" (a form of constructivism) which made him present the six-semester cycle on analysis (from elementary calculus to the Riemann Mapping Theorem!) which I took from 1948 to 1951 based on that approach. So, later on, when topos theory came along, it seemed just a variant of a setting I had been familiar with since the beginning of my studies.
- **C.G.** You have also published in category theory, and categorical techniques pervade many of your papers. Would you agree that this is inevitable given your conceptual vision?

- **B.B.** Absolutely. When I first came in contact with category theory I had the feeling here was something I had always unconsciously expected.
- **C.G.** You have the reputation of being a dedicated and meticulous referee for journals. How important has this been for your own research?
- **B.B.** My activity in this regard did not have very much impact on my own work. I viewed it as a service I happily provided, but the latter was not really influenced by whether I did this or not.
- **C.G.** A colleague described your initial and growing interest in frame theory as a synthesis of your earlier work on topology, order and logic. Is this a fair and/or complete assessment?
- **B.B.** It is certainly very much influenced by my earlier work in topology, less so by that concerning order, and only indirectly connected with logic. Primarily I was drawn to frames by my original interest in formulating topological notions in "purely lattice-theoretic" (and thus in frame) terms. A typical example is the relation  $\prec\prec$ , first introduced in the frame of open sets of a topological space in my 1953 thesis, which replaced a notion Alexandroff had introduced by means of real-valued continuous functions. My first significant encounter of frames was a talk by H. Dowker at the 1966 Topology Symposium in Prague on the Urysohn Lemma for frames.
- C.G. I vividly recall a colleague, known for his somewhat measured and formal demeanour, enthusing about the presentation of one of your lectures —- unusual for him —- even though he had only a little understanding of the subject matter. You are regarded as the consummate stylist, whether in front of a blackboard or in the presentation of your papers or, indeed, in your immaculate and distinctive hand written manuscripts; with a hierarchy of symbols which culminate in beautiful Fraktur capitals. Does this simply reflect a personality trait or do you believe it to be an integral part of your mathematics; or both?
- **B.B.** Regarding the last question, I would say both. Generally, I like to see things, be they thoughts or objects, presented in some well-structured way so that is a personality trait. Mathematics then obviously provides a specific area in which to exercise that tendency, with the

welcome side-effect that greater elegance often means greater clarity. As to the careful handwriting, this evolved mainly to make it easier for secretaries to type a text basically meaningless to them (in the 50s and 60s we had a *faculty* typing pool).

- **C.G.** You appear to never be satisfied with a proof, either your own or another's, when the concepts underlying it are not transparent. Would you in fact take the position that no proof is complete until the underlying concepts are exposed?
- **B.B.** Sorry, but I don't quite understand the question. My feeling with regard to proofs is that, taking the underlying concepts as given, I am concerned that each step in the argument should be presented in the most transparent way possible while the whole sequence of steps should be as economical as possible. However, transparency and economy are ultimately relative notions, it would seem.
- C.G. A large part of your publication output is devoted to frame theory, and there is a noticeable increase in the number of papers produced annually since your retirement! Anyone can see from your publications that you have contributed to an impressive number of areas, and the reader should look at MathSciNet for example (217 listed papers as of today) to see the extent of your interests. What is the thrust of your research at the moment?
- **B.B.** The main thrust is definitely frames. One particular project (with the help of Papiya Bhattacharjee and Joanne Walters-Wayland) involves turning the lecture notes for a course on frames I gave at the University of Cape Town in 1988 into a book an undertaking considerably more demanding than expected. Other than that, there is some joint work going on with A.W. Hager to find a simplification of a very involved proof concerning archimedean  $\ell$ -groups with weak order unit, and some work on my own concerning the rings of integer-valued continuous functions in pointfree topology.
- **C.G.** What advice would you give to a young mathematician when it comes to writing papers?

- **B.B.** Believe it or not, I don't think I have ever given such advice, at least not as a deliberate act, and so I honestly feel I have nothing to say here.
- C.G. You have collaborated with mathematicians from North America, Europe, Africa, the Middle East, the sub-continent and Asia. How important was this for you and, in recent years, how did you manage, given your resistance to email?
- **B.B.** I consider the joint papers with my coauthors a very important, and in some cases absolutely crucial, part of my work, and very much appreciate the experience of this collaboration. Regarding my notorious resistance to e-mail, that really presents no problem here: it is most adequately taken care of by my secretary (many thanks Debbie).
- **C.G.** You were brought up by a single mother who was an innovator of note in the Hamburg school system; do you believe this had an influence in your path to academia?
- **B.B.** My mother's professional activities you refer to only started relatively late in her life, given that the political situation in Germany prevented her from playing any influential rôle during the Hitler period 1933 1945. It is true that she was prominent by the time I started university, but my own academic mindset was formed much earlier than that. What undoubtedly contributed to my development was the fact that she herself had obtained a doctorate (in art history) at the age of 23, and she had a brother and a sister with doctorates. So my own academic orientation was really a family thing. I might add that her father was a Masurian shepherd, and who was deeply committed to providing the best possible education for his children, including (!) his daughters.
- **C.G.** You obtained your PhD at the age of 27. Prior to that you had been conscripted into the war effort at the age of 17 and later you were a prisoner of war for almost two years. How did this influence your career?

**B.B.** I think the only influence was that it delayed the beginning of my university studies by two years. When I returned home in 1947 my aim, unchanged by the delay, was to obtain a doctorate as fast as possible (first intended to be in theoretical physics, but then quickly changed to mathematics), and then to emigrate as soon as I could find suitable employment abroad. As it turned out, I did have the incredibly good fortune to be appointed to an assistantship (the beginning position in a university career) in my department at Hamburg University in November 1953, as a result of a decision by my supervisor, but that did not stop me pursuing my original plan, and so I wound up at Mc-Master University in September 1955, beginning an association that has by now lasted for 61 years.