

# Categories and General Algebraic Structures with Applications

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Dedicated to Bernhard Banaschewski on the Occasion of his 90th Birthday (I)





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# Categories and General Algebraic Structures with Applications

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